Cosmology with Clusters

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Origin of Structure

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but structures exist: LSS, Clusters, Galaxies, stars...

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but $\delta h \ll 1$ so Newton dynamics is enough.

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CMB:

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Minimal asumption: gravity should be active.

The Coma cluster

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3D surveys

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Velocity dispersion in galaxy clusters.

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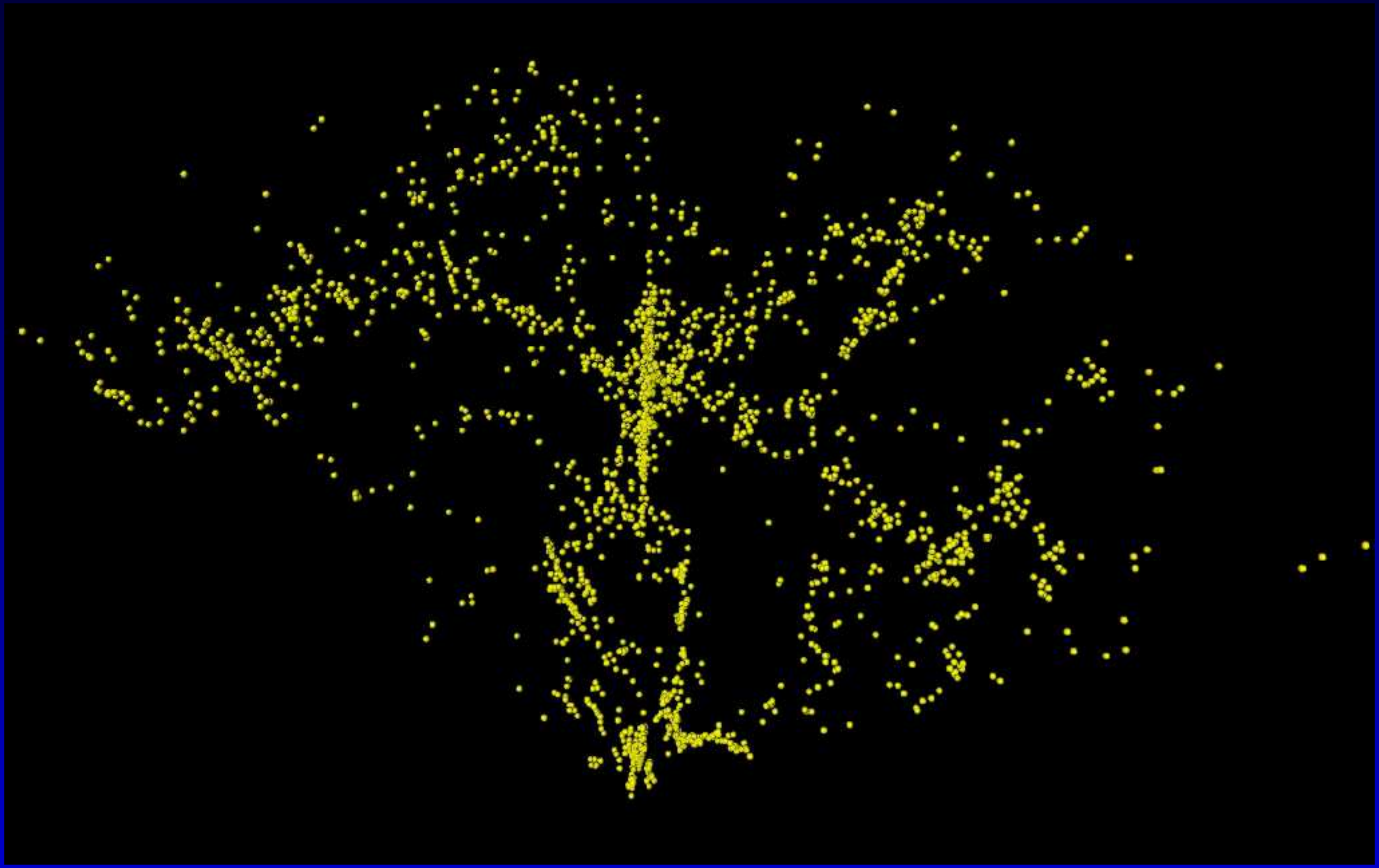

$$v = H_0 D + v_{pec} \cos(\theta)$$

Clusters: a tool for cosmologists

So:

$$D_{obs} = D_{true} + H_0^{-1} V_{pec} \cos(\theta)$$

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Zwicky ($\sim$ 1930) inferred the presence of dark matter.

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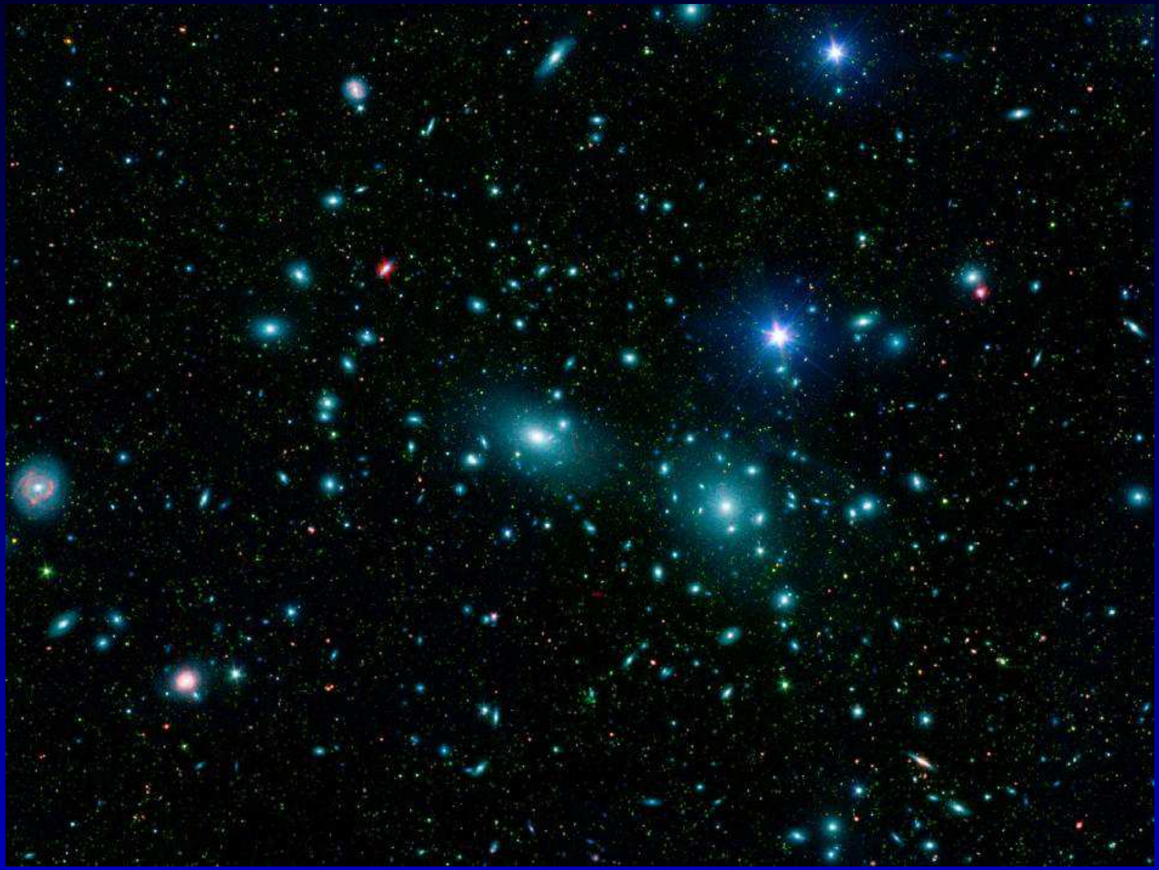

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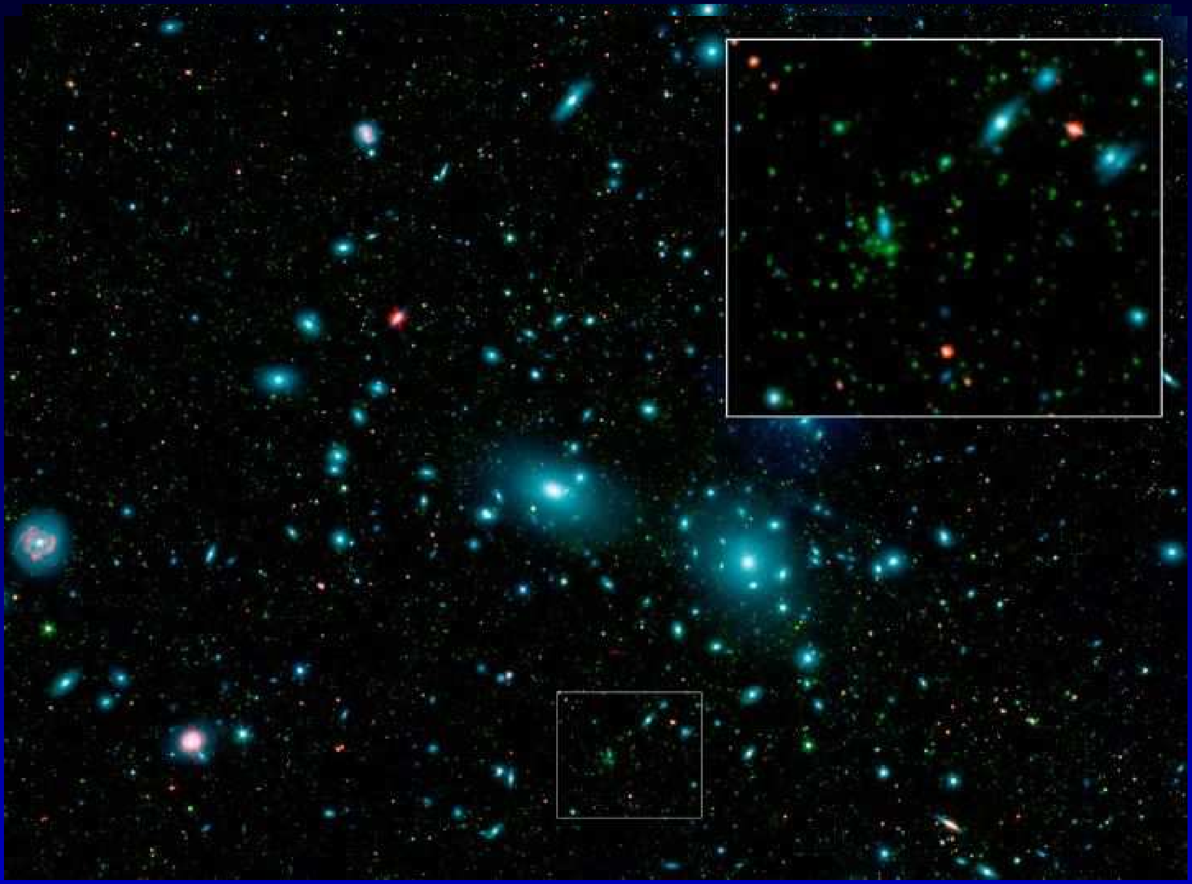

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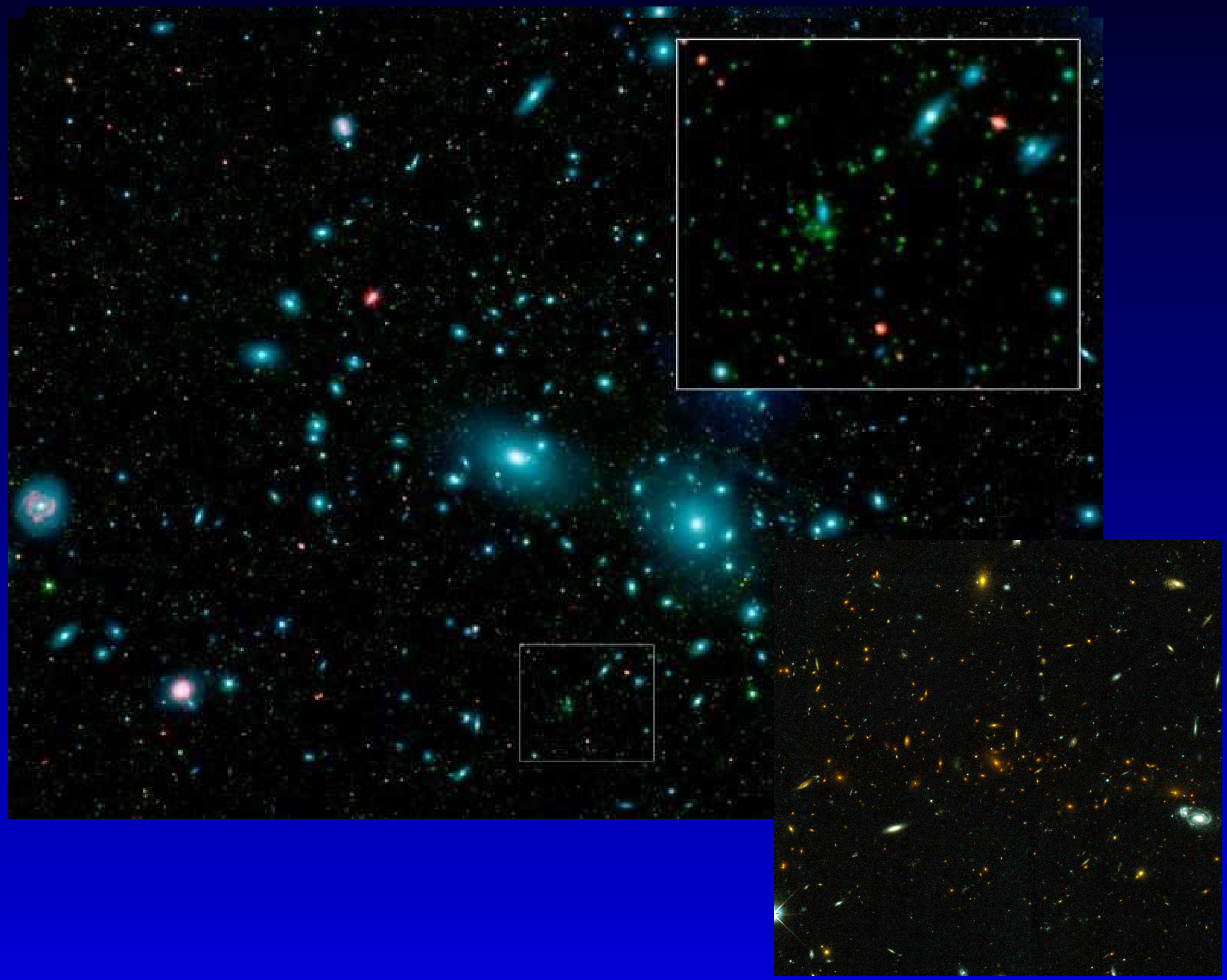

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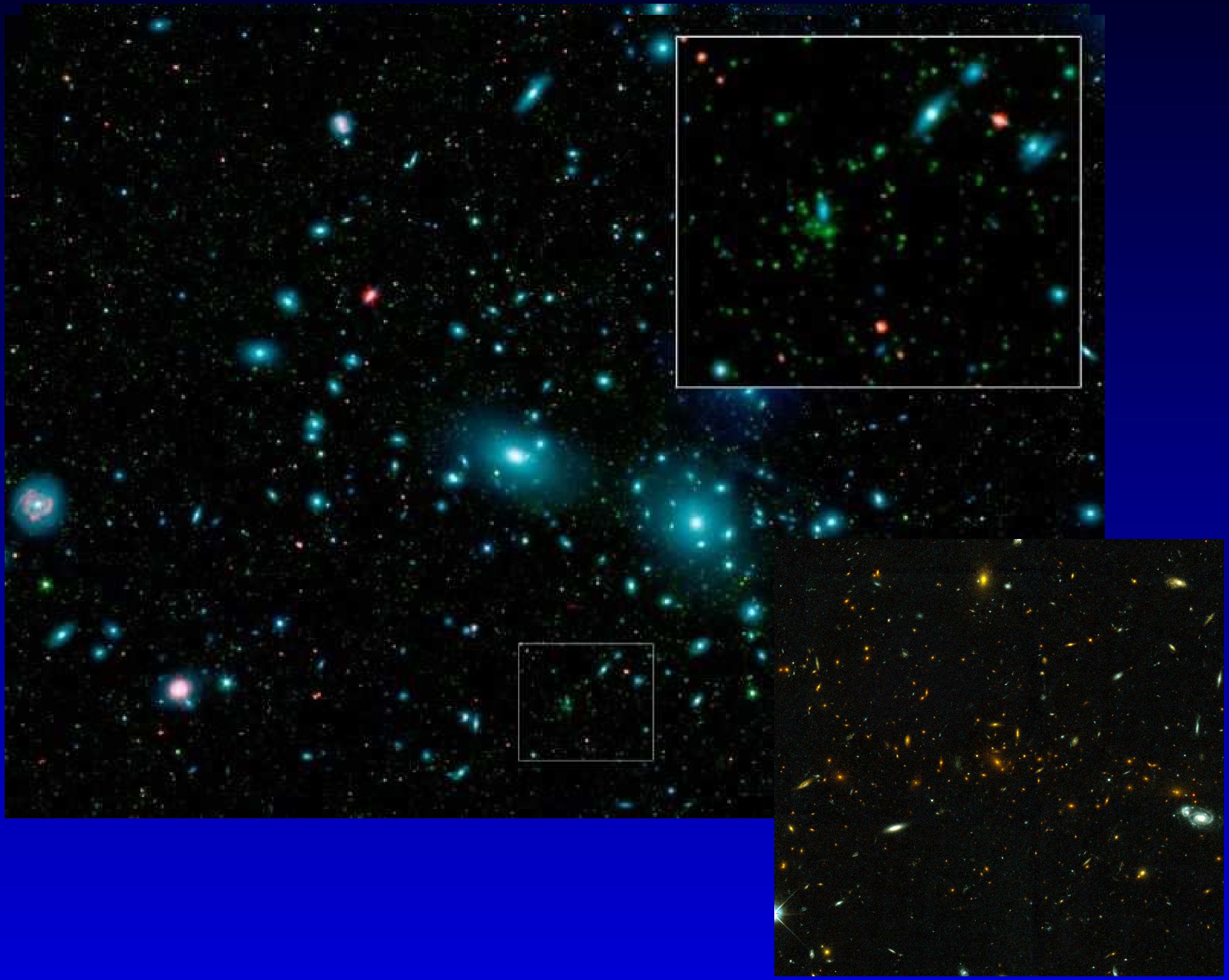

Optical data : Stars, metals, velocity dispersion $\rightarrow$ Mass...

X-ray Visions on clusters

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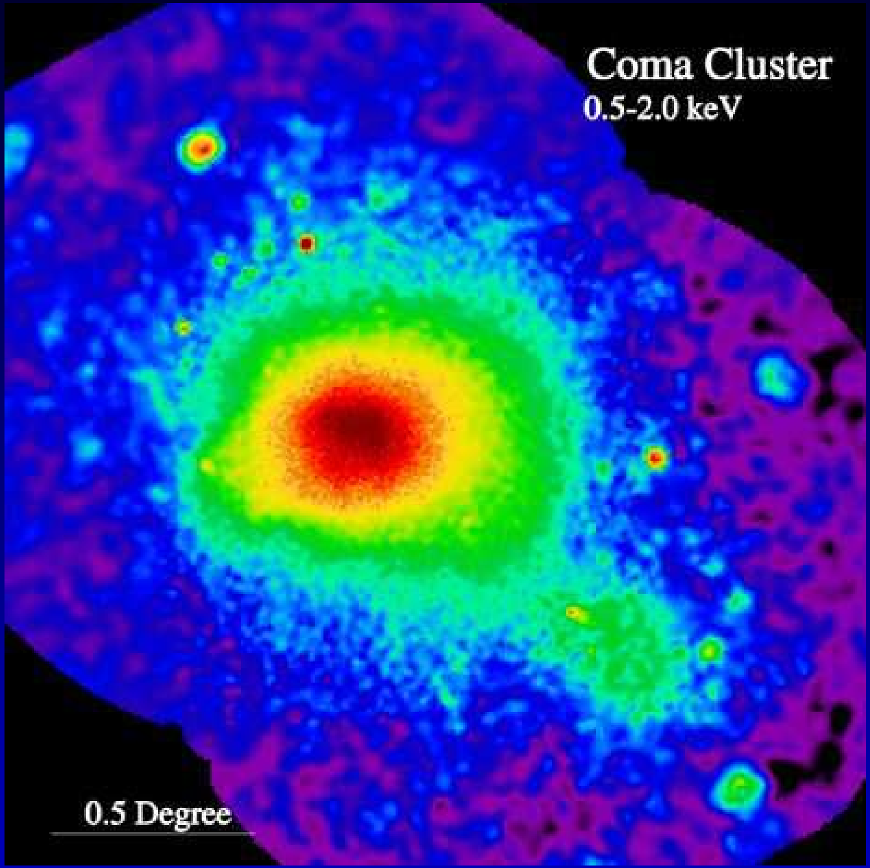

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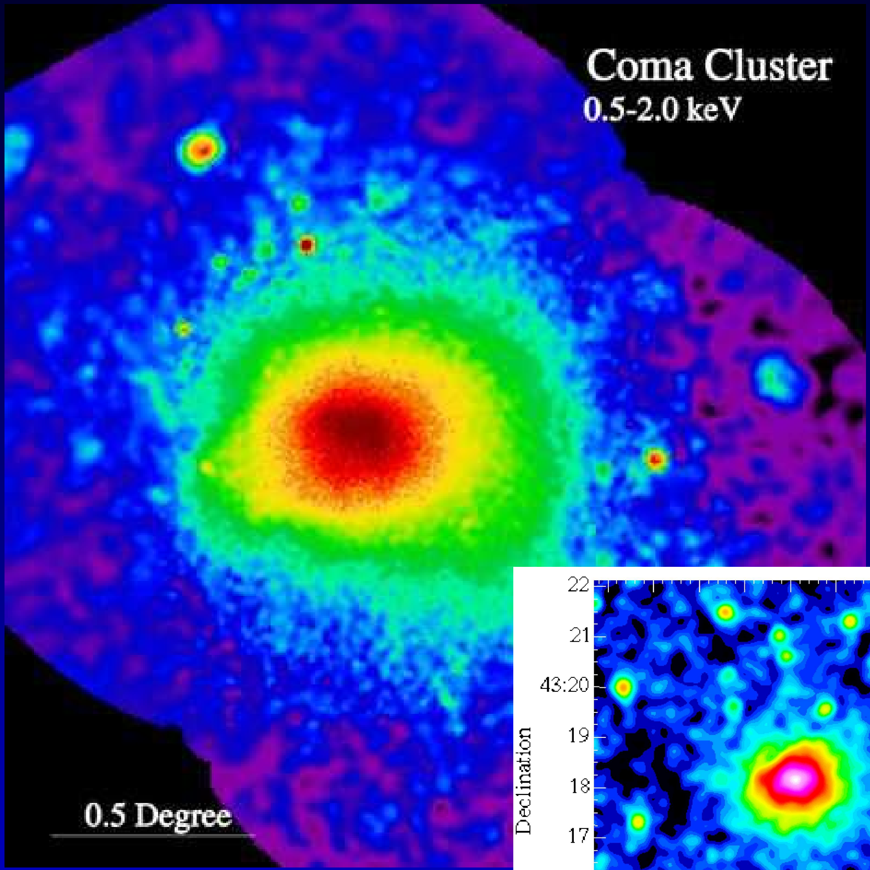

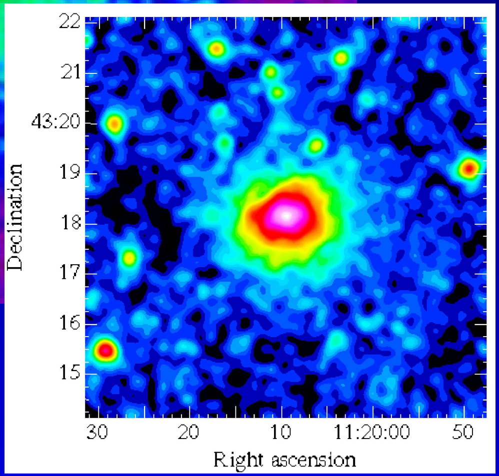

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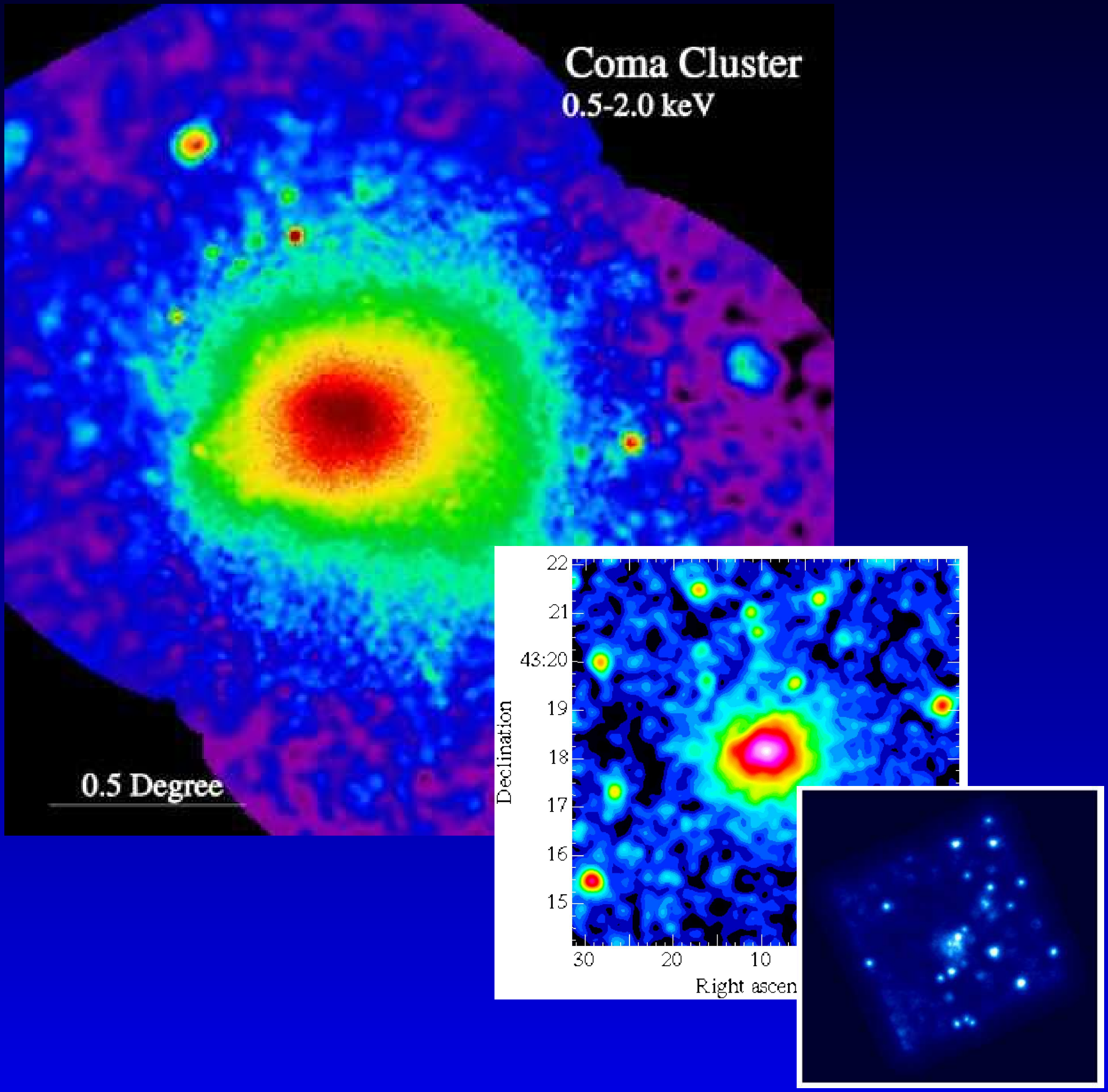

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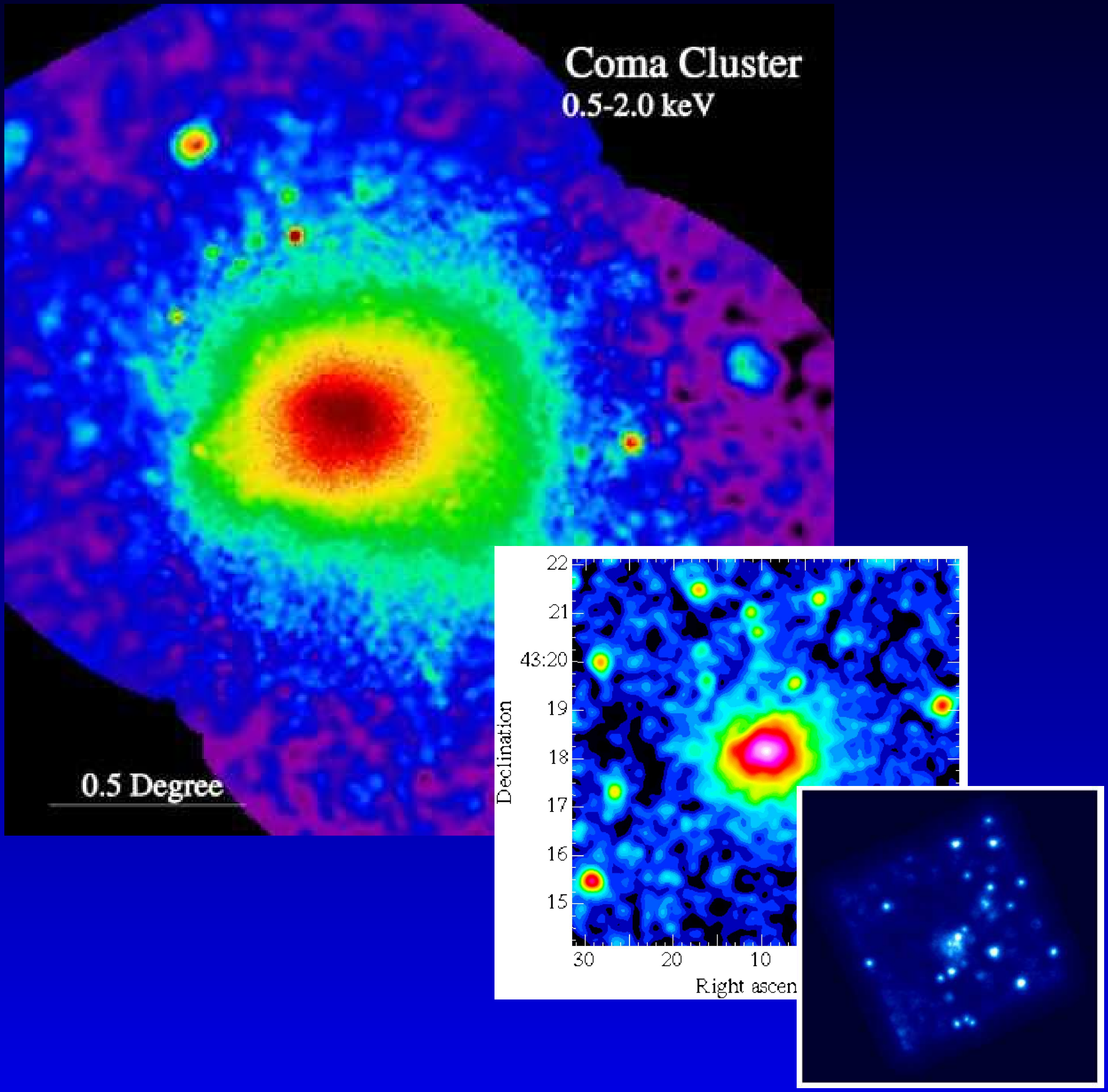

X-ray data : Gas, metals, temperature $\rightarrow$ Mass...

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XMM view of A548b

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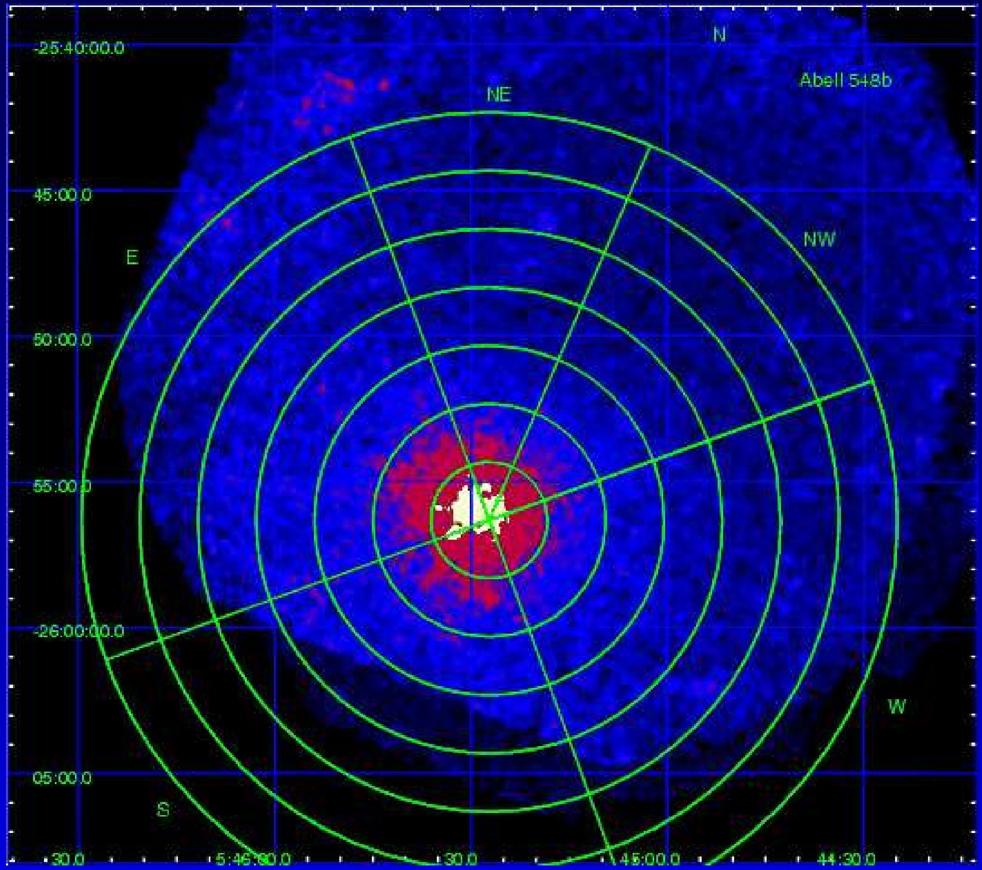

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XMM view of A548b: Sx profile

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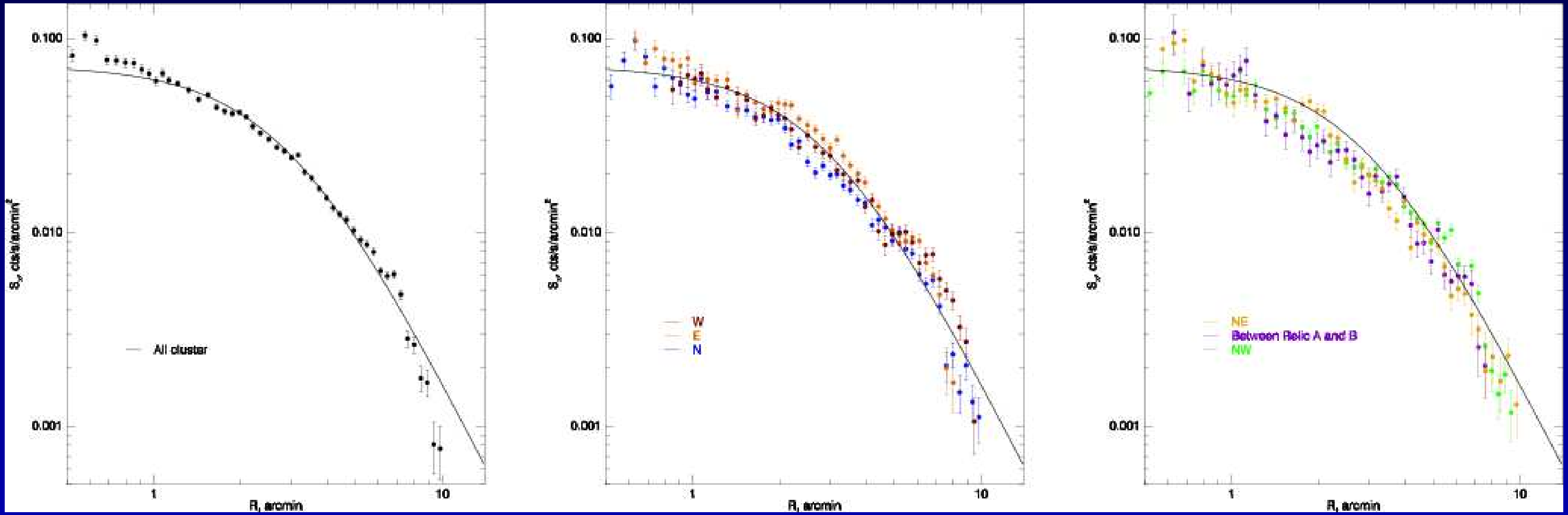

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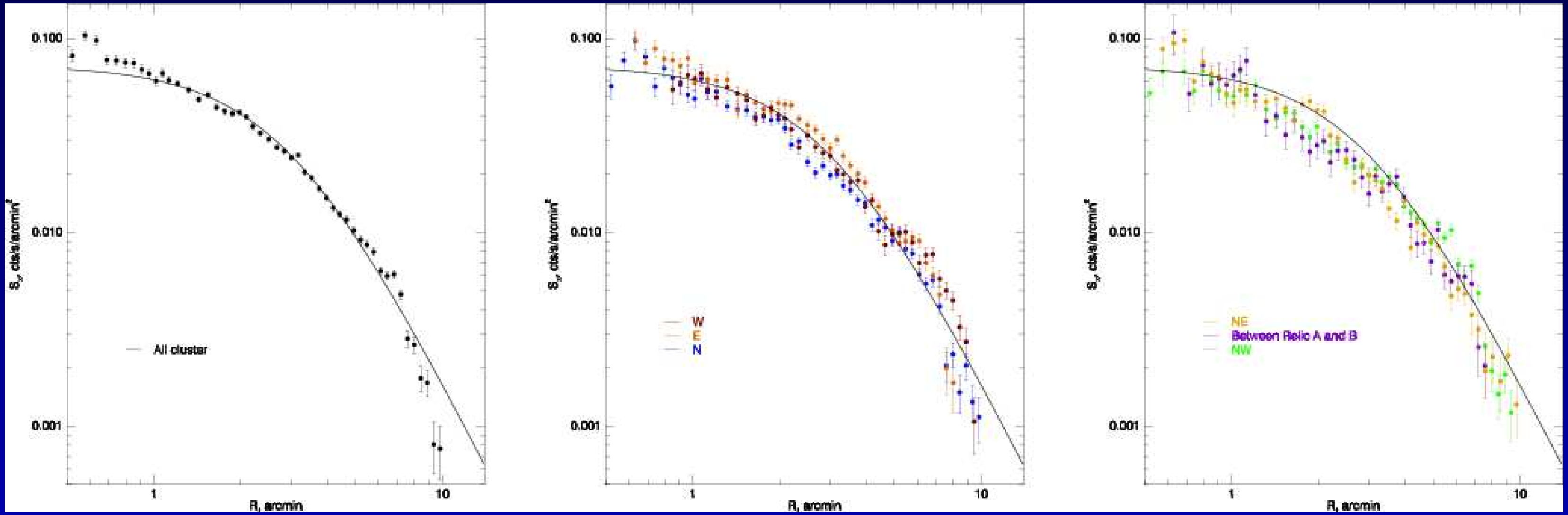

$$s(\theta) = \frac{s_0}{\left(1 + (\theta/\theta_c)^2\right)^{3\beta - 0.5}}$$

Sx profile

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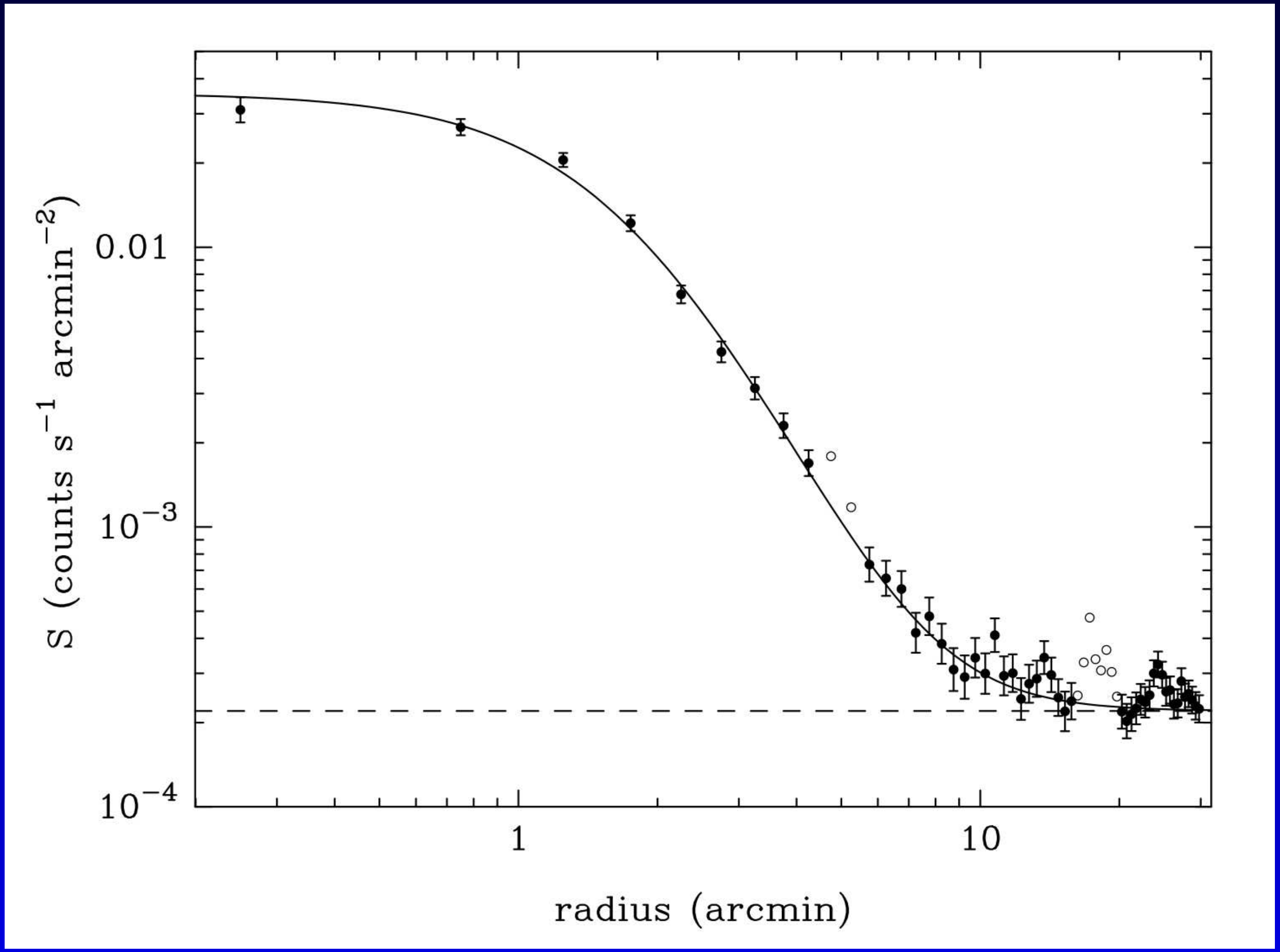
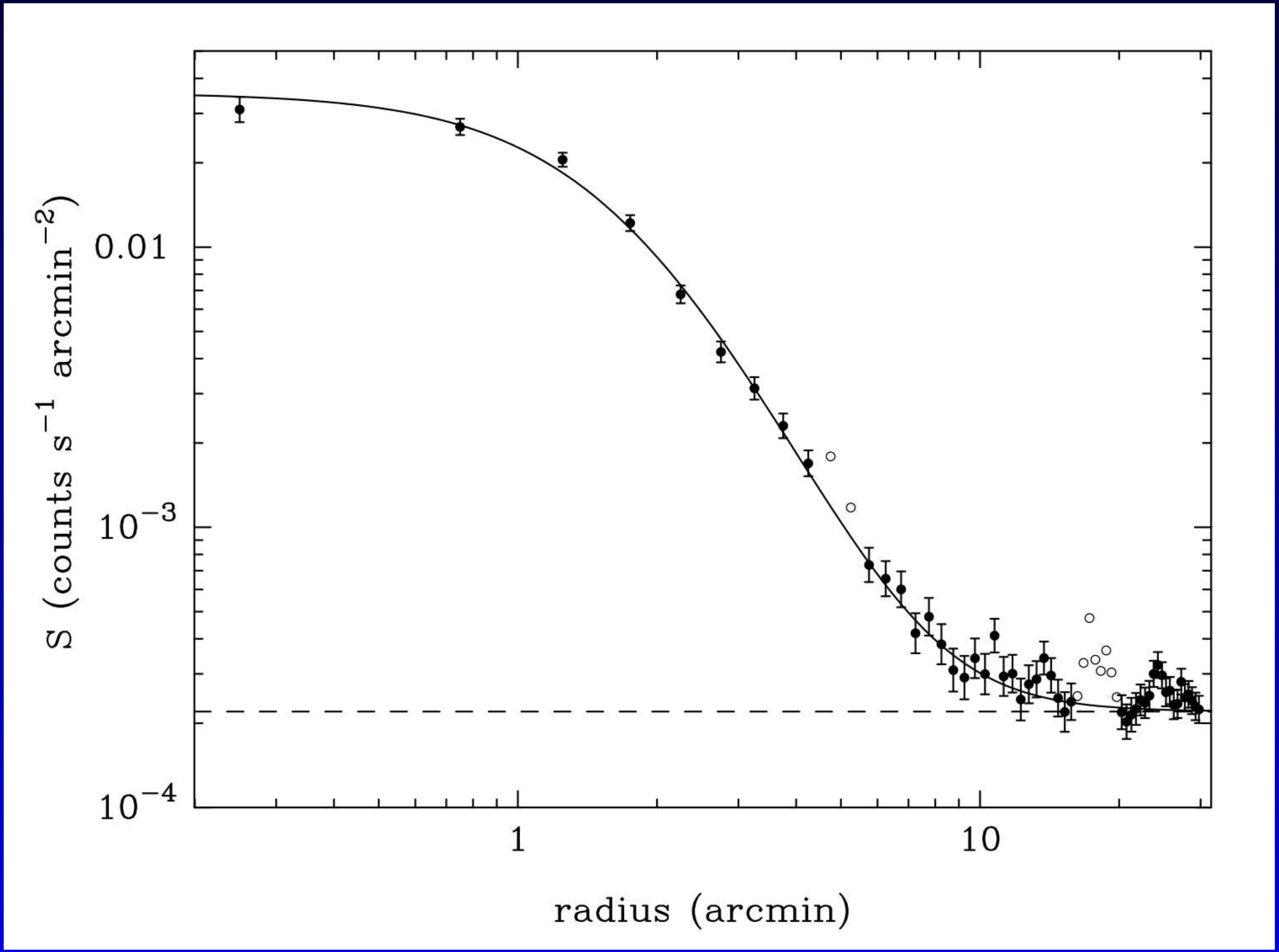


Sx profile



background has to be substracted.

Temperature

The gas in clusters has temperatures between 1 and 10 keV.

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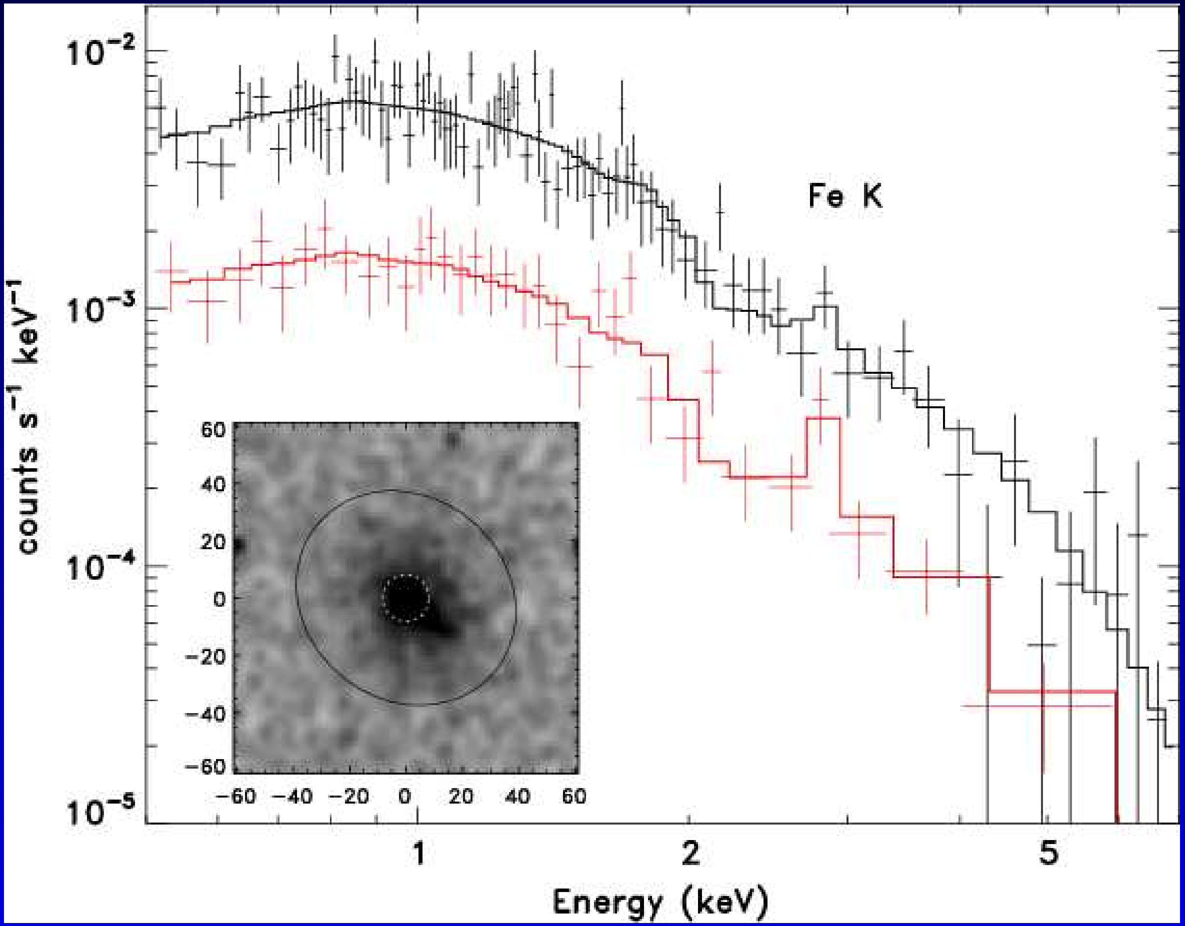

Hydrostatic mass estimates

Hydrostatic equilibrium

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$$\frac{dP_g}{dr} = -\frac{\rho_g G M(r)}{r^2}$$

with $P_g = nkT = \frac{\rho_g}{\mu m_p} kT$

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Isothermal beta model ($r \gg r_c$) :

$$M(r) = -3\beta \frac{kT(r)}{G\mu m_p} r$$

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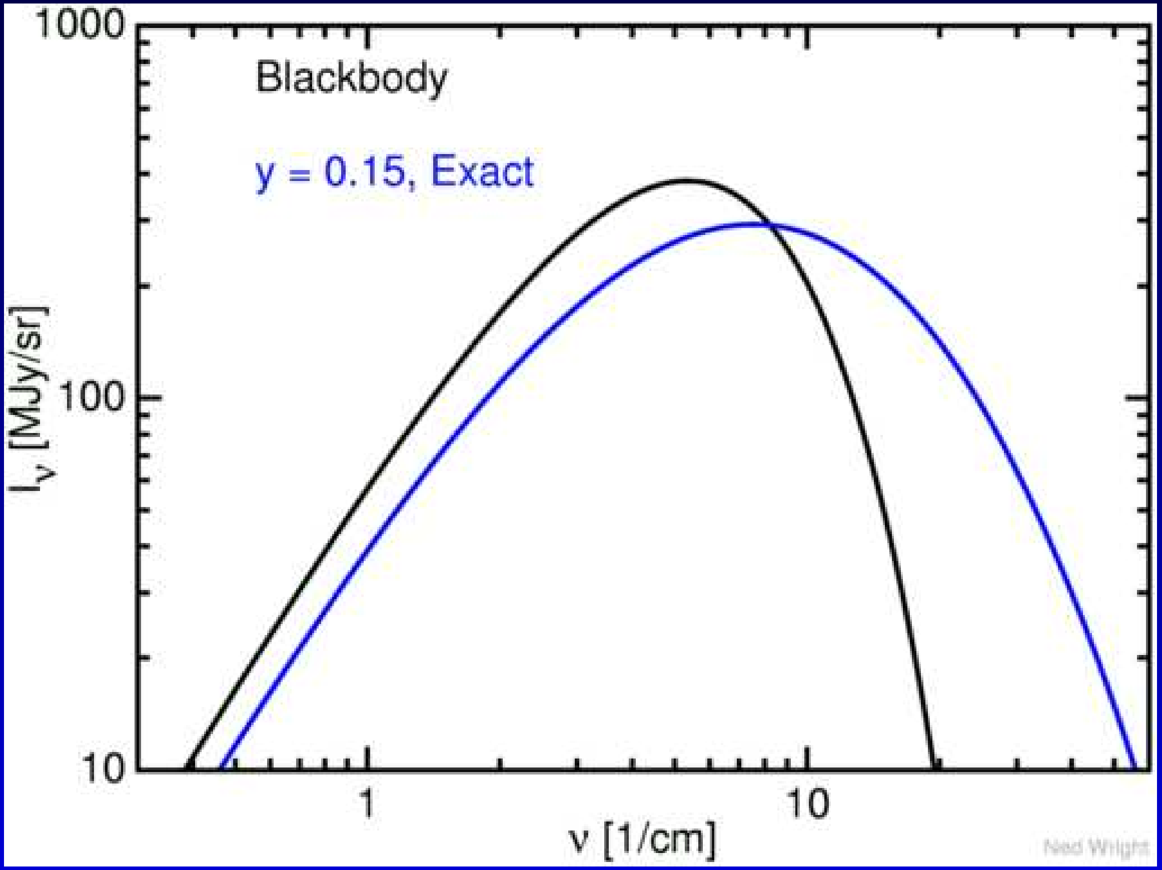

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Sunyaev-Zeldovich effect: induces spectral distorsion.

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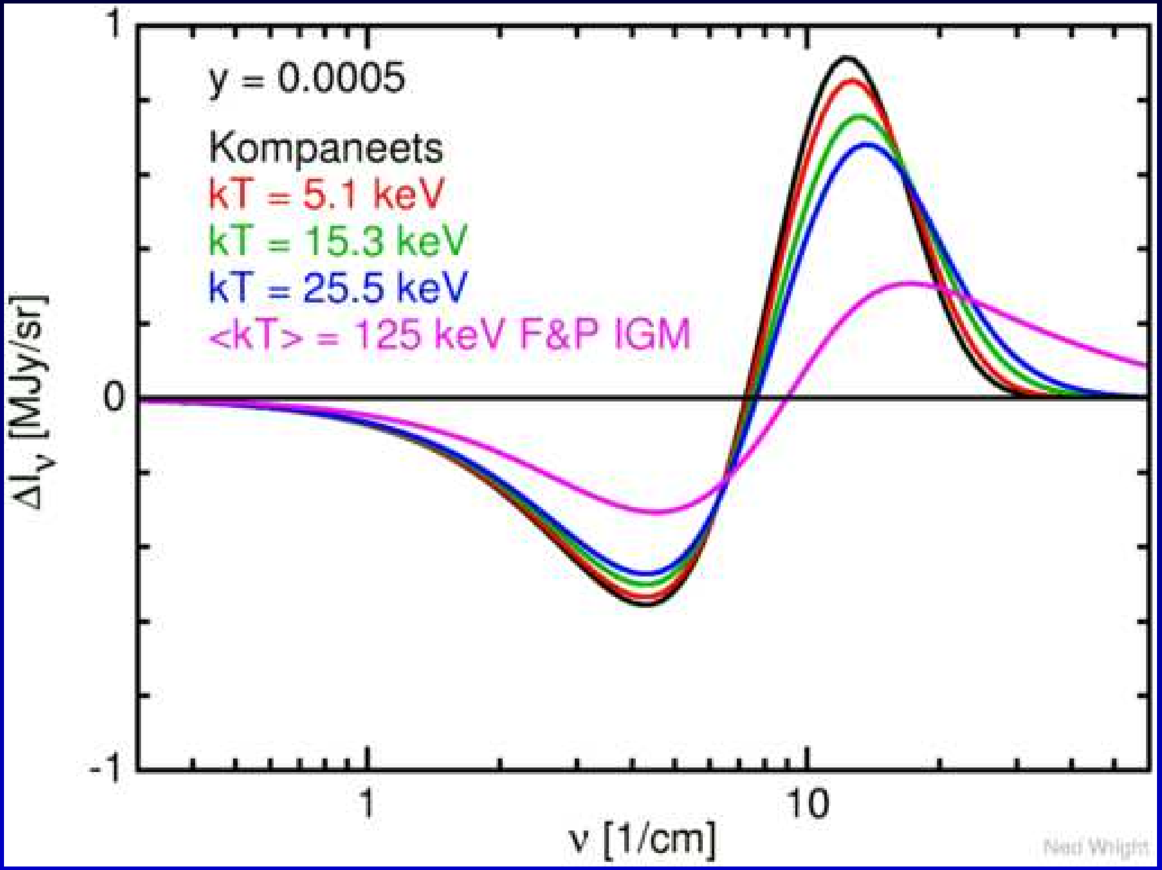

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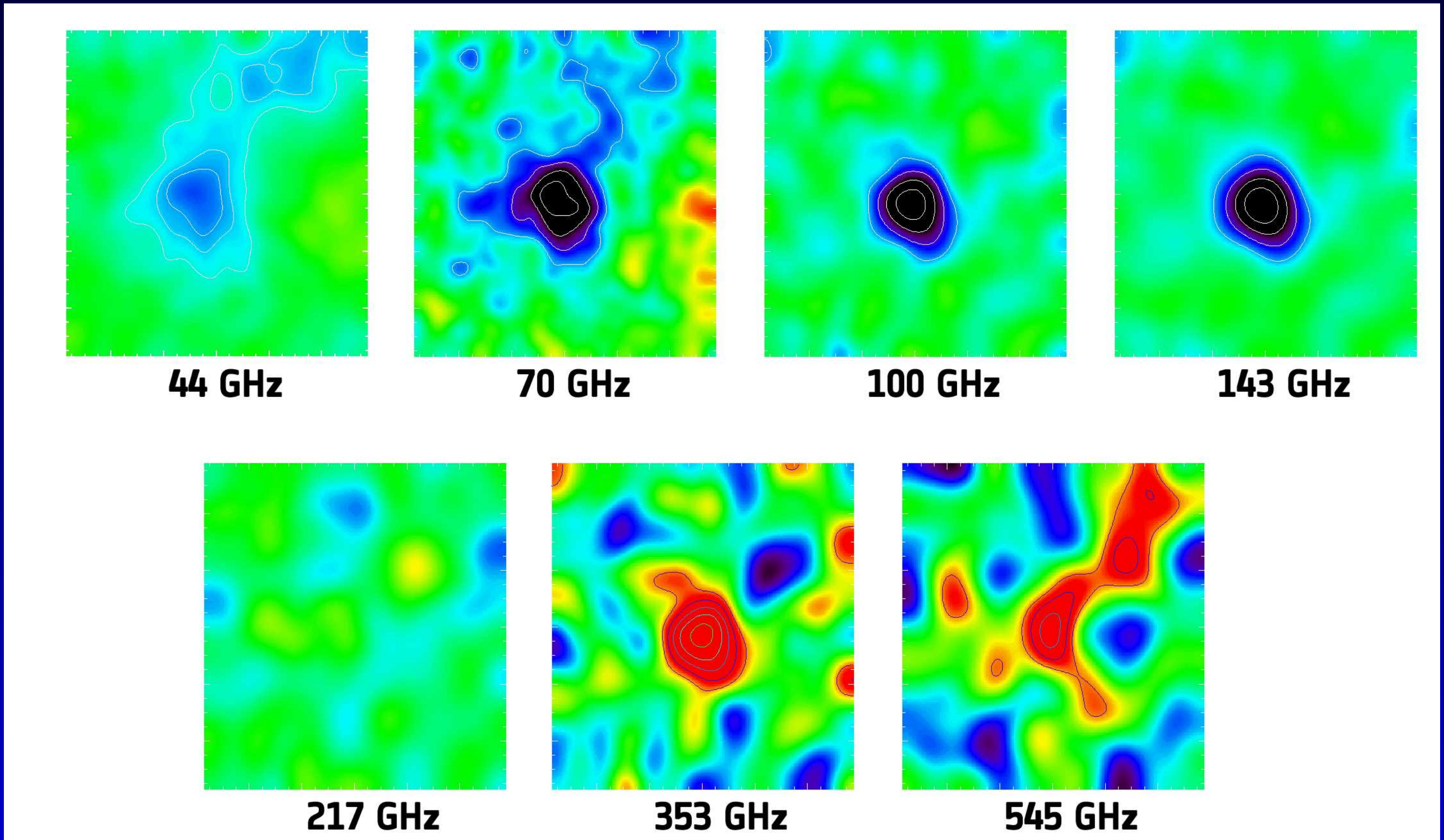
A2319 by Planck



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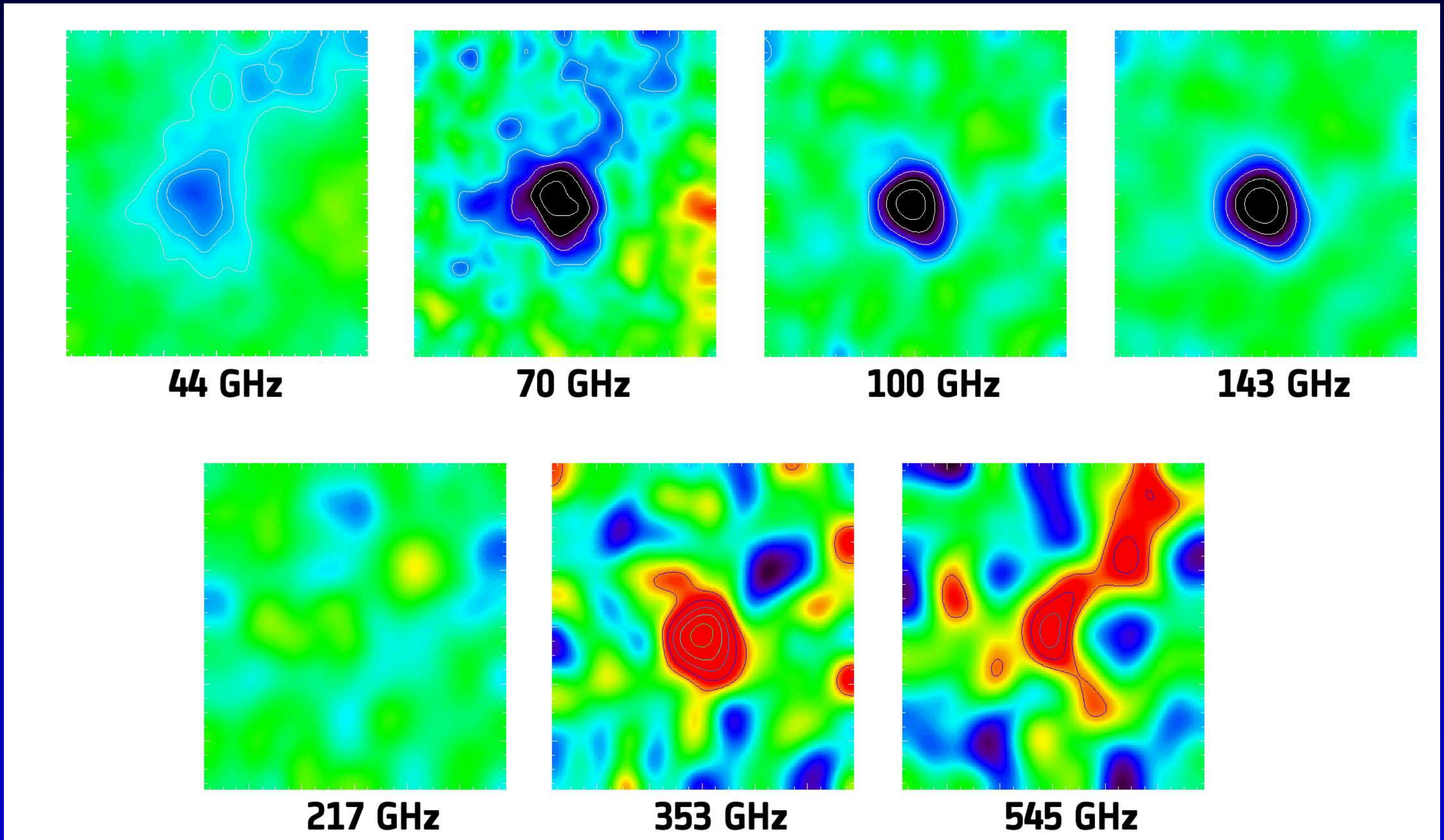

SZ Signal : Gas mass $\times$ temperature $\rightarrow$ Mass...

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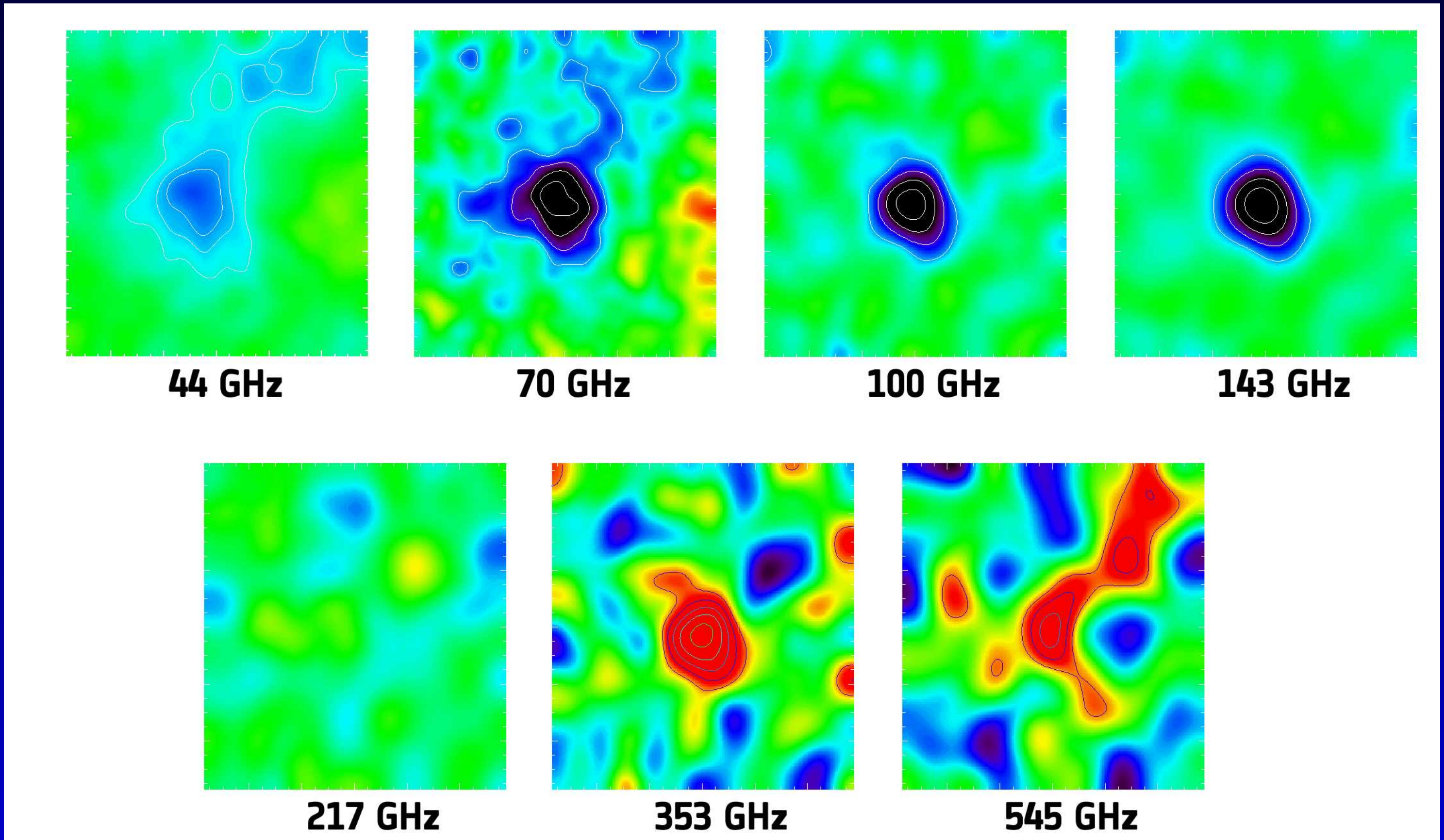

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No dimming with redshift

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- $\rightarrow$ fundamental probes for cosmology

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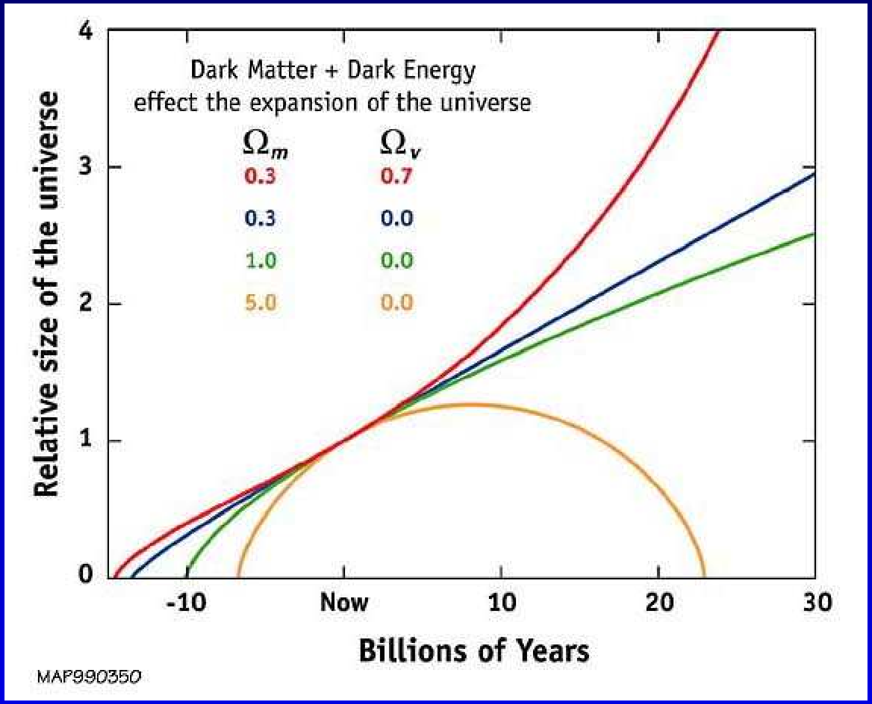

Spherical Perturbation I

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$$\tilde{H}_0 \, t = \frac{\tilde{\Omega}_0}{2(\tilde{\Omega}_0 - 1)^{3/2}}(\phi - \sin(\phi))$$

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$$\Delta_m = \frac{9}{16}\pi^2 - 1. \simeq 4.55$$

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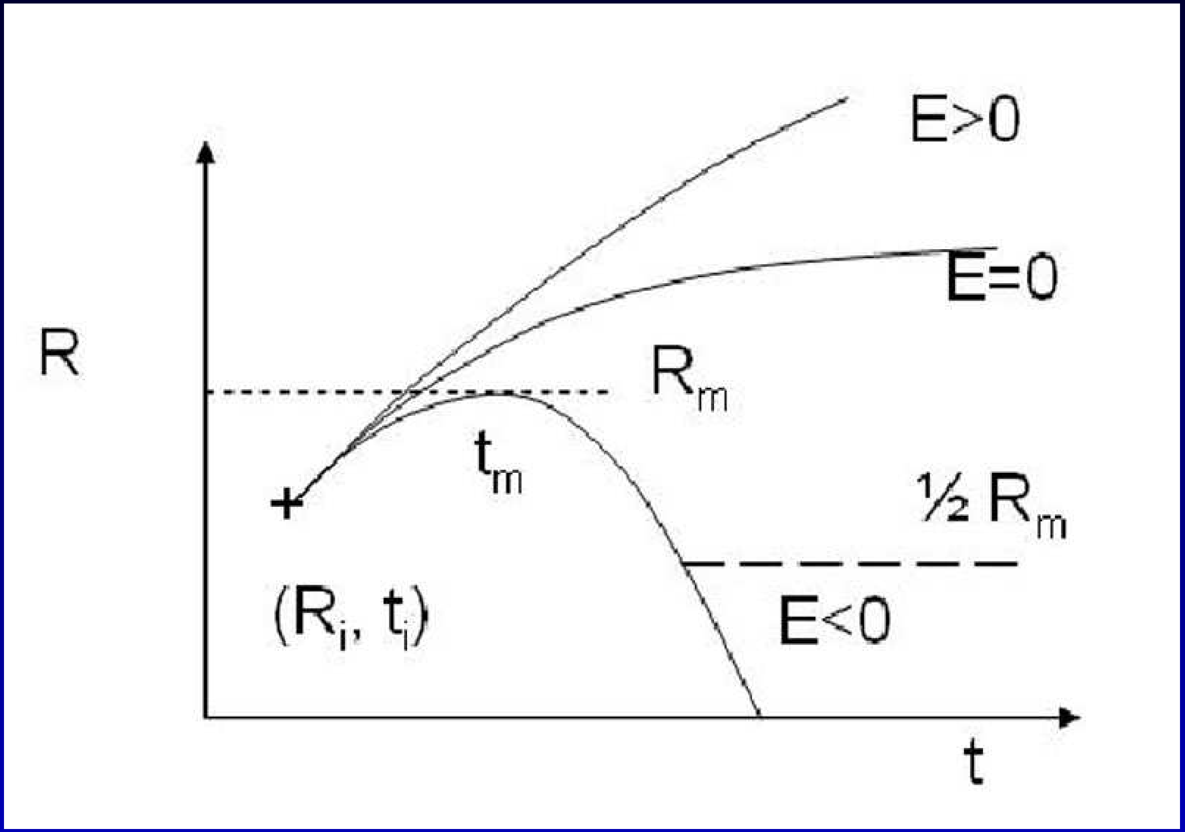

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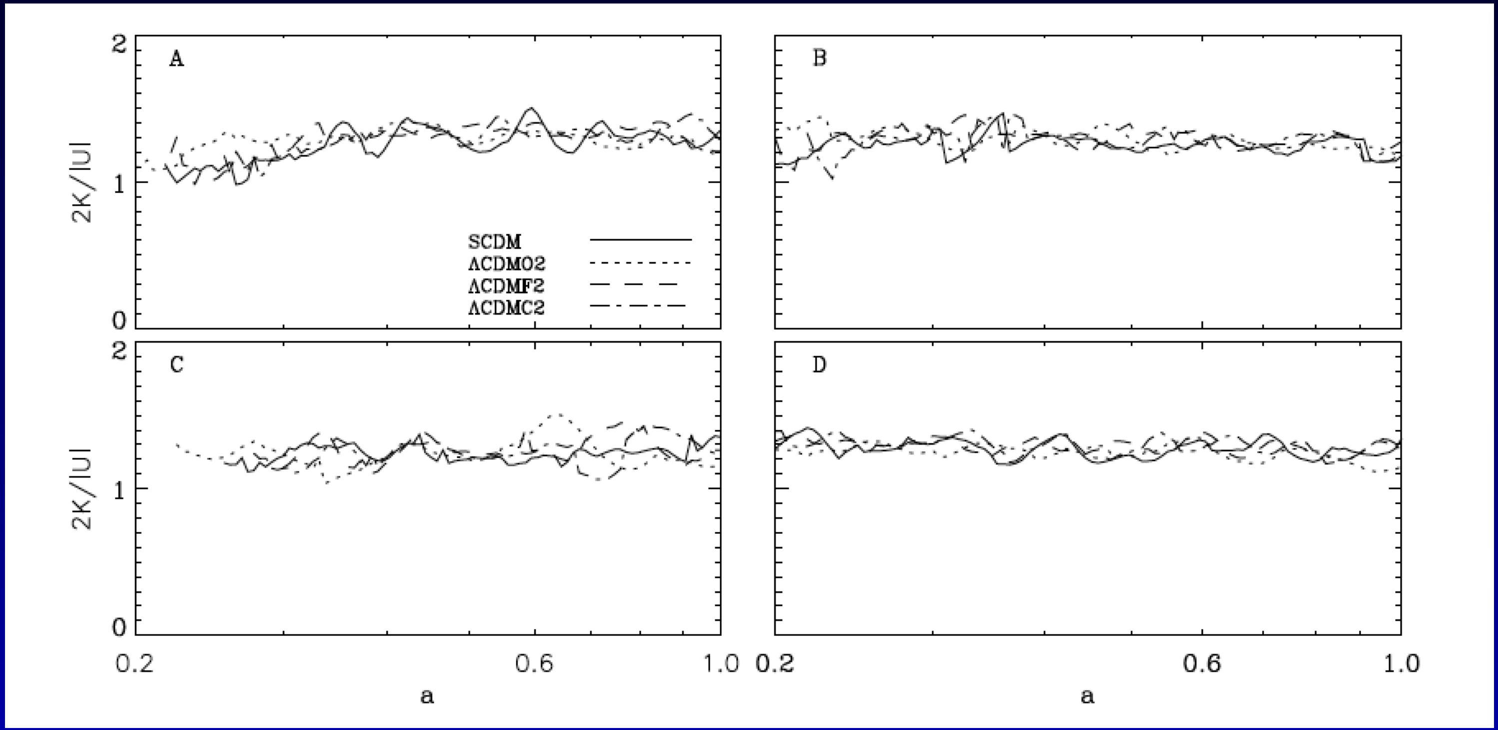

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let's estimate the linear expected amplitude at viriliza-tion.

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$$\tilde{\rho} = \frac{8\rho_m}{(1-\cos\psi)^3} = \frac{64\rho_m}{\psi^6(1-\psi^2/4)}$$

$$t = \frac{t_m}{\pi}(\psi - \sin\psi) = \frac{t_m}{\pi}\frac{\psi^3}{6}[1 - \frac{\psi^2}{20}]$$

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so:

$$\psi^6 = \left(\frac{6\pi t}{t_m}\right)^2 [1 + \frac{\psi^2}{10}]$$

Virialization IV

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and

$$\tilde{\rho} = (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10})\frac{64\rho_m t_m^2}{(6\pi)^2 t^2}$$

$$= (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10})\frac{64}{36\pi^2}\frac{3\pi^2}{32\pi G t^2} = \rho(1 + \frac{3\psi^2}{20})$$

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so with : $\tilde{\rho} = \rho(1+\delta)$

$$\delta = \frac{3}{20}\psi^2 = \frac{3}{20}\left(\frac{6\pi t}{t_m}\right)^{2/3} = \frac{3(6\pi)^{2/3}}{20}\frac{1+z_m}{1+z}$$

Conclusion

(for the spherical collapse model)

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$$\delta_m = \frac{3(6\pi)^{2/3}}{20}(1+z_m) = 1.06(1+z_m)\text{when}\Delta_m \simeq 4.5$$

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$$\delta_m = 2^{2/3}\frac{3(6\pi)^{2/3}}{20}(1+z_v) = 1.68(1+z_m)\text{when}\Delta_v \simeq 177.$$

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Transition into the non linear regime is extremely rapid.

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$$\delta_m = \frac{3(6\pi)^{2/3}}{20}(1+z_m) = 1.06(1+z_m)\text{when}\Delta_m \simeq 4.5$$

and

$$\delta_m = 2^{2/3}\frac{3(6\pi)^{2/3}}{20}(1+z_v) = 1.68(1+z_m)\text{when}\Delta_v \simeq 177.$$

Transition into the non linear regime is extremely rapid.

For $z < z_v$, $\Delta = 177 \left(\frac{1+z_v}{1+z}\right)^3$

Flat cosmology

(for the spherical collapse model)

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$$\delta_{th} = \frac{3}{20}(12\pi)^{2/3}[1 + 0.0123\log(\Omega(z)].$$

(Kitayama and Suto, 1996)