Cosmology with Clusters

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Origin of Structure

Big Bang: homogeneous distribution
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but structures exist: LSS, Clusters, Galaxies, stars...
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Basic scheme:

Primordial Universe: $\frac{\delta \rho}{\rho} \ll 1$

Gravitational instability scenario.
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Gravitational instability scenario.

Present universe: \( \frac{\delta \rho}{\rho} \gg 1, \xi_g(r), \phi(L) \)
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Primordial Universe: \( \frac{\delta \rho}{\rho} \ll 1 \)
Gravitational instability scenario.

Present universe: \( \frac{\delta \rho}{\rho} \gg 1, \xi_g(r), \phi(L) \)
but \( \delta h \ll 1 \) so Newton dynamics is enough.
Fluctuations of the metric

galaxies: $V_{rot} \sim 50 - 300$ km/s
Fluctuations of the metric

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$$\delta h \sim \frac{G \delta M}{rc^2} \sim \frac{V^2}{c^2} \sim 10^{-6}$$
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as such:

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for a cluster: \( \sigma \sim 500 - 2000 \text{km/s} \)

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\delta h \sim \frac{\sigma^2}{c^2} \sim 10^{-5}
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**CMB:**

$$\frac{\delta T}{T} \sim \delta h \sim \frac{\sigma^2}{c^2} \sim 10^{-5}$$
...and RW metric

For large scale structures:
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validity of RW cannot be tested by LSS... but by CMB!
Gravitational instability
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1900 Jeans: static medium, exponential growth:

\[ \tau \sim \frac{1}{\sqrt{G\rho}} \]
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Gravitational instability

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Lemaître 1933, exact spherical solution with R.G. Lipshitz 1946, linear theory, growth \( \propto t^n \)

Minimal assumption: gravity should be active.
The Coma cluster
The Coma cluster
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3D surveys
3D surveys

Velocity dispersion in galaxy clusters.
3D surveys

Velocity dispersion in galaxy clusters.

\[ v = H_0 D + v_{pec} \cos(\theta) \]
Clusters: a tool for cosmologists

So:

\[ D_{\text{obs}} = D_{\text{true}} + H_0^{-1} V_{\text{pec}} \cos(\theta) \]
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Measures \[ \sigma_{1D} = \sigma_{3D} / \sqrt{3} \]
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Isothermal sphere:

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Zwicky (~1930) inferred the presence of dark matter.
Clusters: a tool for cosmologists
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Optical data: Stars, metals, velocity dispersion → Mass...
X-ray Visions on clusters
X-ray Visions on clusters

Coma Cluster
0.5-2.0 keV

0.5 Degrees
X-ray Visions on clusters
X-ray Visions on clusters
X-ray Visions on clusters

X-ray data: Gas, metals, temperature → Mass...
X-ray Visions on clusters

XMM view of A548b
X-ray Visions on clusters

XMM view of A548b
X-ray Visions on clusters

XMM view of A548b: Sx profile
X-ray Visions on clusters

XMM view of A548b: Sx profile
X-ray Visions on clusters

XMM view of A548b: Sx profile

\[ s(\theta) = \frac{s_0}{\left(1 + \left(\frac{\theta}{\theta_c}\right)^2\right)^{3\beta - 0.5}} \]
Sx profile
Sx profile

![Graph showing S vs radius](image-url)
Sx profile

background has to be substracted.
Temperature

The gas in clusters has temperatures between 1 and 10 keV.
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Hydrostatic mass estimates

Hydrostatic equilibrium
Hydrostatic mass estimates

Hydrostatic equilibrium

\[
\frac{dP_g}{dr} = -\frac{\rho_g GM(r)}{r^2}
\]

with \( P_g = nkT = \frac{\rho_g}{\mu m_p} kT \)
Hydrostatic mass estimates

Hydrostatic equilibrium

\[ \frac{dP_g}{dr} = -\frac{\rho_g GM(r)}{r^2} \]

with \( P_g = nkT = \frac{\rho_g}{\mu m_p} kT \) leading to:

\[ M(r) = \frac{kT(r)}{G\mu m_p} \left( \frac{d\ln \rho_g}{d\ln r} + \frac{d\ln T}{d\ln r} \right) r \]
Hydrostatic mass estimates

Hydrostatic equilibrium

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Isothermal beta model \((r \gg r_c)\):

\[ M(r) = -3 \beta \frac{k T(r)}{G \mu m_p} r \]
SZ Vision on clusters
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Sunyaev-Zeldovich effect: induces spectral distortion.
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A2319 by Planck
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\[ \text{SZ Signal: Gas mass } \times \text{ temperature } \rightarrow \text{ Mass...} \]
SZ Vision on clusters
A2319 by Planck

**SZ Signal**: Gas mass $\times$ temperature $\rightarrow$ Mass...
No dimming with redshift
Final words
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Clusters are unique objects in astrophysics:
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• Baryons content can be measured/estimated
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• in redundant ways
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Clusters are unique objects in astrophysics:

- Baryons content can be measured/estimated
- Metals content can be estimated
- Mass content can be estimated
- in redundant ways
- → fundamental probes for cosmology
Non linear regime
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General problem very complex
Non linear regime

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1-dimensional approximation allows analytical calculations.
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Spherical model (Lemaître, 1933)
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Newtonian problem.
Non linear regime

General problem very complex
1-dimensional approximation allows analytical calculations.
Spherical model (Lemaître, 1933)
Newtonian problem. Solution already seen:

![Graph showing the effect of Dark Matter and Dark Energy on the expansion of the universe. The graph plots the relative size of the universe against billions of years. The Ω_m and Ω_v values are shown for different scenarios.](image)
Spherical Perturbation I
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\[ \tilde{H}_0 t = \frac{\tilde{\Omega}_0}{2(\tilde{\Omega}_0 - 1)^{3/2}} (\phi - \sin(\phi)) \]

\[ R(t) = \frac{\tilde{\Omega}_0 \tilde{R}_0}{2(\tilde{\Omega}_0 - 1)} (1 - \cos(\phi)) \]
Spherical Perturbation I

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Density at maximum:

\[ \tilde{\rho} = \tilde{\rho}_0 \left( \frac{\tilde{R}_0}{\tilde{R}} \right)^3 \]
Spherical Perturbation II
Spherical Perturbation II

At maximum: $\tilde{R}_m \leftrightarrow \phi = \pi$
Spherical Perturbation II

At maximum: \( \tilde{R}_m \leftrightarrow \phi = \pi \)

\[ \tilde{\rho}_m = \frac{3\tilde{H}_0^2}{32\pi G} \frac{4(\tilde{\Omega}_0 - 1)^3}{\tilde{\Omega}_0^2} \]

\[ \tilde{H}_0 t_m = \frac{\tilde{\Omega}_0}{2(\tilde{\Omega}_0 - 1)^{3/2}} \pi \]
Spherical Perturbation II

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i.e.

$$\tilde{\rho}_m = \frac{3\pi^2}{32\pi G t_m^2}$$

with: $1 + \Delta_m = \frac{\tilde{\rho}_m}{\rho}$ and $\rho = \frac{1}{6\pi G t^2}$ (EdS)
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\[
\Delta_m = \frac{9}{16} \pi^2 - 1 \approx 4.55
\]
Virialization I
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At $2t_m$ solution reaches a singularity.
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At $2t_m$ solution reaches a singularity. During collapse kinetic energy prevents singularity.
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At $2t_m$ solution reaches a singularity. During collapse kinetic energy prevents singularity. Initially:

$$K = 0 \text{ and } U_i = -\frac{\alpha GM^2}{R_i}$$
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In the final stage (virialization):

$$K_f = -\frac{1}{2}U_f$$
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so:

$$R_f = \frac{1}{2}R_i$$
Virialization: I
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Virialization II
Virialization II

Contrast density at virialization:

\[ 1 + \Delta_v = \frac{9}{16} \pi^2 \]
Virialization II

Contrast density at virialization:

\[ 1 + \Delta_v = \frac{9}{16} \pi^2 \times 2^3 \]
Virialization II

Contrast density at virialization:

\[ 1 + \Delta_v = \frac{9}{16} \pi^2 \times 2^3 \times (2^{2/3})^3 \]
Virialization II

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\[ 1 + \Delta_v = \frac{9}{16} \pi^2 \times 2^3 \times (2^{2/3})^3 = 18\pi^2 \approx 178 \]
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let’s estimate the linear expected amplitude at virilization.
Virialization III
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\[ \delta(z) = \delta_0 (t/t_0)^{2/3} = \frac{\delta_0}{1 + z} \]

\( \delta_0 \) linear amplitude today.
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\( \delta_0 \) linear amplitude today.

\[ \tilde{\rho} = \frac{8\rho_m}{(1 - \cos \psi)^3} = \frac{64\rho_m}{\psi^6 \left(1 - \frac{\psi^2}{4}\right)} \]

\[ t = \frac{t_m}{\pi} (\psi - \sin \psi) = \frac{t_m \psi^3}{\pi} \left[1 - \frac{\psi^2}{20}\right] \]
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so:

\[ \psi^6 = \left( \frac{6\pi t}{t_m} \right)^2 [1 + \frac{\psi^2}{10}] \]
Virialization IV
Virialization IV

and

\[ \tilde{\rho} = (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10}) \frac{64 \rho_m t_m^2}{(6\pi)^2 t^2} \]

\[ = (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10}) \frac{64}{36\pi^2} \frac{3\pi^2}{32\pi G t^2} = \rho \left(1 + \frac{3\psi^2}{20}\right) \]
Virialization IV

and

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so with : \( \tilde{\rho} = \rho \left(1 + \delta\right) \)

\[ \delta = \frac{3}{20} \psi^2 = \frac{3}{20} \left(\frac{6\pi t}{t_m}\right)^{2/3} = \frac{3(6\pi)^{2/3}}{20} \frac{1 + \alpha_m}{1 + z} \]
Conclusion
(for the spherical collapse model)
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(for the spherical collapse model)

\[ \delta_m = \frac{3(6\pi)^{2/3}}{20}(1+z_m) = 1.06(1+z_m) \text{ when } \Delta_m \simeq 4.5 \]

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Conclusion
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\[ \delta_m = \frac{3(6\pi)^{2/3}}{20} (1+z_m) = 1.06(1+z_m) \text{ when } \Delta_m \approx 4.5 \]

and

\[ \delta_m = 2^{2/3} \frac{3(6\pi)^{2/3}}{20} (1+z_v) = 1.68(1+z_m) \text{ when } \Delta_v \approx 177. \]
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Transition into the non linear regime is extremely rapid.
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Transition into the non linear regime is extremely rapid.

For \( z < z_v \), \( \Delta = 177 \left( \frac{1 + z_v}{1 + z} \right)^3 \)
Flat cosmology

(for the spherical collapse model)
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(for the spherical collapse model)

\[ \Delta_v \approx 18\pi^2 + 82x - 39x^2 \]

(compared to \( \rho_c(z) \)) with \( x = \Omega(z) - 1 \) (Bryan and Norman, 1998) and
Flat cosmology
(for the spherical collapse model)

\[ \Delta_v \approx 18\pi^2 + 82x - 39x^2 \]

(compared to \( \rho_c(z) \)) with \( x = \Omega(z) - 1 \) (Bryan and Norman, 1998) and

\[ \delta_{th} = \frac{3}{20}(12\pi)^{2/3} [1 + 0.0123 \log(\Omega(z))]. \]

(Kitayama and Suto, 1996)