Cosmology with Clusters

Alain Blanchard

alain.blanchard@irap.omp.eu





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Scaling principle

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so that M and z are the only two numbers to characterize a cluster (Δ is set by the cosmology...or by the cosmologist!)

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Assume isothermal distribution:

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with
$$\langle v^2 \rangle = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\sigma^2$$

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 so

$$\sigma = \sqrt{\frac{GM}{r}}$$

Velocity dispersion III Numerically:

$$\sigma = 1130(hM_{15})^{1/3} \left(\frac{\Delta\Omega_m}{178}\right)^{1/6} \sqrt{1+z} \text{ km/s}$$

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Scaling laws (dependence on mass and redshift).

Numerically: good agreement with numerical simulations (Bryan and Norman, 1998):



 $\frac{1}{2}\mu m_p V^2 = \frac{3}{2}kT$

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This is the scaling of the M - T relation.

Temperature scaling law

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 $L_x \propto n^2 V T^{1/2}$

leading to :

 $Lx \propto M^{4/3} (1+z)^{7/2} (\Omega_m \Delta/178)^{1/6}$

Inspired from Press and Schechter (1974) The density field $\rho(x)$ has to be smoothed:

$$\tilde{\delta}(x) = \int \delta(x+u) W_R(u) du$$

and

$$\overline{\tilde{\delta}^2(x)} = \sigma^2(R)$$

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For a top hat window (!):

$$M(R) = \frac{4\pi}{3}R^3\overline{\rho}$$

Let's $s(\delta)$ be the probability that a volume element dV will be included in a NL object with mass greater than M given that it is included in a fluctuation of radius > R with a contrast density δ .

$$\int_{M}^{+\infty} mn(m)dm = \overline{\rho} \int \mathcal{F}_{\delta}(\delta)s(\delta)d\delta$$

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$$\int_{M}^{+\infty} mn(m)dm = \overline{\rho} \int \mathcal{F}_{\delta}(\delta)s(\delta)d\delta$$

for a sharp threshold i.e. $\delta > \delta_{NL}$:

$$\int_{M}^{+\infty} mn(m)dm = \overline{\rho} \int_{\delta_{NL}}^{+\infty} \mathcal{F}_{\delta}(\delta)d\delta = \overline{\rho} \int_{\nu_{NL}}^{+\infty} \mathcal{F}(\nu)d\nu$$

Following the spherical model:

$$\nu_{NL} = \frac{\delta_{NL}}{\sigma(M)}$$

Just derive against M:

$$n(M) = -\frac{\rho}{M^2 \sigma(M)} \delta_{NL} \frac{\ln \sigma}{\ln M} \mathcal{F}(\nu_{NL})$$

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and test it against numerical simulations...

But...

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It actually works! (Borgani et al., 2000)

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e-PS formula

ST (Sheth & Tormen, 1999) expression for \mathcal{F} (S.O) :

$$\mathcal{F}(\nu) = \sqrt{\frac{2a}{\pi}} A \exp(-0.5a\nu^2)(1. + 1./(a\nu^2)^p)$$

with a = 0.707 A = 0.3222 p = 0.3.
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Allows to investigate structure formation.



Jenkins et al. (2001)

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Jenkins et al. (2001)



Jenkins et al. (2001) Different fit:

$$f(M) = \alpha \exp\left(-|\ln(\sigma^{-1}) + \beta|^{\gamma}\right)$$

set constant threshold ($\delta_{NL} = 1.686$) and constant contrast density.



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Universal mass function ?



Tinker et al. (2008)

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Tinker et al. (2008) Different fit...



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Non-universal mass function ?



Despali et al. (2015)



Despali et al. (2015) Different fit back to ST formula... a = 0.7689 A = 0.3222 p = 0.3



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An other fit on cluster scales...

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Despali et al. (2015)



Despali et al. (2015) Universal mass function 5-7%.

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- Allows to investigate structure formation. History of individual structure is missing: merging tree → semi-analytical method "SAM" in order to model galaxy formation : assembly/evolution.
- Warning: data come through "light" which is coming from baryons and this was almost not discussed in these lectures...

Important progresses are due to numerical simulations:

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20 years ago...

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| TABLE 1 Simulation Parameters | | | | | | | | | | |
|-----------------------------------------|------------------------------|--------------------------|------------------------------|---------------------------|--------------------------|-----------------------------|------------------------------------------------------------------------------|--------------------------------------------------------------|----------------------|--|
| Designation | $\Omega_{\rm cold}$ | $\Omega_{\rm hot}$ | $\Omega_{\rm baryon}$ | h | $(eV)^{m_v}$ | σ_8 | $N_{\rm cell}$ | $N_{\rm part}$ | $(h^{-1} Mpc)$ | |
| CDM270 CHDM512 OCDM256 CHDM256 | 0.94 0.725 0.34 0.6 | 0.0 0.2 0.0 0.3 | 0.06 0.075 0.06 0.1 | 0.5 0.5 0.65 0.5 | 0 2 × 2.3 0 7.0 | 1.05 0.7 0.75 0.67 | 270 ³ 512 ³ 256 ³ 256 ³ | $135^{3} \\ 3 \times 256^{3} \\ 128^{3} \\ 3 \times 128^{3}$ | 85 50 85 85 | |

2015...

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2015...

| Main set of simulations | | | | | | | | | |
|-------------------------|---------------------|-------|------------------------|----------------------|------------------|------------------|---------|--|--|
| name | box [h^{-1} Mpc] | z_i | $m_p[M_{\odot}h^{-1}]$ | soft [kpc h^{-1}] | $N_{h-tot}(z=0)$ | $N_{h>300}(z=0)$ | colour | | |
| Ada | 62.5 | 124 | $1.94 	imes 10^7$ | 1.5 | 2264847 | 103852 | green | | |
| Bice | 125 | 99 | $1.55	imes10^8$ | 3 | 2750411 | 129674 | cyan | | |
| Cloe | 250 | 99 | $1.24 	imes 10^9$ | 6 | 3300880 | 161580 | blue | | |
| Dora | 500 | 99 | $9.92 	imes 10^9$ | 12 | 3997898 | 191793 | magenta | | |
| \mathbf{Emma} | 1000 | 99 | $7.94 	imes 10^{10}$ | 24 | 4739379 | 176633 | red | | |
| Flora | 2000 | 99 | $6.35 	imes 10^{11}$ | 48 | 5046663 | 75513 | orange | | |

Table 1. Features of Le SBARBINE simulations run with Planck13 parameters $\Omega_m = 0.307$, $\Omega_{\Lambda} = 0.693$, $\sigma_8 = 0.829$ and h = 0.677 and containing 1024^3 dark matter particles. The last two columns report the total number of haloes identified with the Spherical Overdensity at redshift z = 0 that are resolved with more than 10 and 300 particles, respectively.

| Secondary set of simulations | | | | | | | | | |
|----------------------------------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|--|--|--|
| name | Ω_m | Ω_{Λ} | σ_8 | box $[h^{-1}Mpc]$ | $m_p[M_{\bigodot}h^{-1}]$ | colour | | | |
| Tea Tea-big Tina Tina-big Vera Vera-big | $0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4$ | 0.8 0.8 0.8 0.8 0.6 0.6 | 0.7 0.7 0.9 0.9 0.7 0.7 | 150 1000 150 1000 150 1000 | $\begin{array}{c} 1.396\times10^9\\ 4.135\times10^{11}\\ 1.396\times10^9\\ 4.135\times10^{11}\\ 2.791\times10^9\\ 8.271\times10^{11}\end{array}$ | gray-square gray-square gray-triangle gray-triangle brown-square brown-square | | | |
| Viola Viola-big Wanda (wmap7) Wanda-big (wmap7) | 0.4 0.4 0.272 0.272 | 0.6 0.6 0.728 0.728 | 0.9 0.9 0.81 0.81 | 150 1000 150 1000 | $\begin{array}{l} 2.791 \times 10^9 \\ 8.271 \times 10^{11} \\ 1.898 \times 10^9 \\ 5.624 \times 10^{11} \end{array}$ | brown-triangle brown-triangle blue-circle blue-circle | | | |

Table 2. Details of the small set of 10 simulations with different cosmological parameters. Each contains 512^3 dark matter particles with initial conditions generated at redshift z = 99. For all the models the Hubble parameter is h = 0.6777, apart from the WMAP7 cosmology for which h = 0.704.

Clusters Self-similarity from simulations:

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 $\sigma(M_*) \sim 1$

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NFW profiles

From numerical simulations DM halo appear to be well fitted by the so-called NFW profile:

$$\frac{\rho(r)}{\rho_c} = \frac{\delta_c}{(r/r_c)(1.+r/r_c)^2}$$

Two parameters: mass in some radius (for instance $\Delta = 200$) and one parameter: the concentration c: $r_c = r_{200}/c$

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allows analytical M(r)

Recent simulations of Clusters:

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Millenium simulation: much more detailled pictures...











here

https://www.youtube.com/watch?v=xfgDoExbu_Q

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Let's first define clusters... From previous pictures, it is not clear... By convention, clusters are defined as regions with contrast density above some threshold:

$$\frac{<\rho_c>}{\rho_r} > 1 + \Delta_{th}$$

Which geometry (spheres, friend-of-friend, ...) ? Which reference density (ρ_r) ? $\rho_u(z)$, $\rho_c(z)$ Which reference contrast (Δ_{th}) ? Δ_v , 178, 200, 500, 2000...

Different halo finders

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Knebe et al. (2013)

$$\delta_m = \frac{3(6\pi)^{2/3}}{20} (1+z_m) = 1.06(1+z_m) \text{when} \Delta_m \simeq 4.5$$

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Transition into the non linear regime is extremely rapid. Can be generalized to other models:

$$\delta_{NL}(z,C), \Delta_{NL}(z,C)$$

Plus profile (M, z, C)...

$$N(M, z) = -\frac{\rho}{m^2 \sigma(M)} \delta_s \frac{d \log \sigma}{d \log M} \mathcal{F}(\frac{\delta_s}{\sigma(M)})$$

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estimation of $\sigma(M) \leftrightarrow P(k)$:

$$\sigma^2(R) = \int P(k)\hat{W}(kR)d^3k$$

with :

$$\hat{W}(kR) = \frac{3\sin(kR)/kR - \cos(kr)}{(kR)^2}$$

Cluster mass function evolution

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 $\sigma(R,z) = D(z)\sigma(R,0)$

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So $\sigma(M, z)$) contains the **growing rate** of fluctuations.

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N(M)dM = N(T)dT
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N(M)dM = N(T)dT

Needs a flux limited survey...

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Each cluster as a luminosity s_x , a redshift z and a T_x

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For each cluster one can compute the volume of detection of the cluster V_i .

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$$N(>T) = \sum_{T_i > T} \frac{1}{V_i}$$

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$$N(>T) = \sum_{T_i > T} \frac{1}{V_i}$$

Unbiased estimator...

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Application to the x-ray temperature:

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so that:

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Fitting $N(T_x)$

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From 50 X-ray cluters (2000)

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Fitting $N(T_x)$



From 72 X-ray cluters (2015)

Measuring local matter fluctuations:

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Evrard et al (2002), Pierpaoli et al. (2003), Seljak (2002), Vauclair et al. (2003), Viana et al. (2003)

Measuring local matter fluctuations:



Evrard et al (2002), Pierpaoli et al. (2003), Seljak (2002), Vauclair et al. (2003), Viana et al. (2003) Consistency and degeneracy...

Back to luminosity scaling law:

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$$Lx \propto M^{4/3} (1+z)^{7/2} (\Omega_m \Delta/178)^{1/6}$$

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 $Lx \propto M^{4/3} (1+z)^{7/2} (\Omega_m \Delta/178)^{1/6}$ leads to $L_x \propto T^2$ While observations indicate to $L_x \propto T^3$!

Back to luminosity scaling law:

 $Lx \propto M^{4/3}(1+z)^{7/2}(\Omega_m \Delta/178)^{1/6}$ leads to $L_x \propto T^2$ While observations indicate to $L_x \propto T^3$! Gas in clusters needs extra heating and scaling law is not expected to hold for L_x (not with M and thereby not with z).



Scaling of the gas content:

Scaling of the gas content:



Scaling of the gas content:



So clusters may be self-similar after all...

(1)
$$Y = K M_g T_g D_a^{-2}$$

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Independent of the gas geometry, clumping, ...

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Independent of the gas geometry, clumping, ... Better mass proxy (Barbosa al., 1997)



(Kravtsov, Vikhlinin, Nagai 2006)

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$$Y = KM_gT_gD_a^{-2}$$

Independent of the gas geometry, clumping, ... Better mass proxy (Barbosa al., 1997) Leading to the scaling law

$$Y = \kappa \xi A_{TM} f_B M^{5/3} h^{8/3} \left(\Omega_M \frac{\Delta(z, \Omega_M)}{178} \right)^{1/3} (1+z) D^{-2}$$

where $\kappa = 1.816.10^{-4}$ and ξ accounts for the difference between T_x and T_g .



The analysis of Planck CMB data provides high precision constraints on (most) cosmological parameters.

Table 4. Parameter 68 % confidence limits for the base Λ CDM model from *Planck* CMB power spectra, in combination with lensing reconstruction ("lensing") and external data ("ext," BAO+JLA+ H_0). Nuisance parameters are not listed for brevity (they can be found in the *Planck Legacy Archive* tables), but the last three parameters give a summary measure of the total foreground amplitude (in μK^2) at $\ell = 2000$ for the three high- ℓ temperature spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN (posterior mean $Y_P \approx 0.2453$, with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on $\Omega_b h^2$).

| Parameter | TT+lowP 68 % limits | TT+lowP+lensing 68 % limits | TT+lowP+lensing+ext 68 % limits | TT,TE,EE+lowP 68 % limits | TT,TE,EE+lowP+lensing 68 % limits | TT,TE,EE+lowP+lensing+ex 68 % limits |
|-------------------------|------------------------|--------------------------------|------------------------------------|------------------------------|--------------------------------------|-----------------------------------------|
| $\Omega_{\rm b}h^2$ | 0.02222 ± 0.00023 | 0.02226 ± 0.00023 | 0.02227 ± 0.00020 | 0.02225 ± 0.00016 | 0.02226 ± 0.00016 | 0.02230 ± 0.00014 |
| $\Omega_{\rm c}h^2$ | 0.1197 ± 0.0022 | 0.1186 ± 0.0020 | 0.1184 ± 0.0012 | 0.1198 ± 0.0015 | 0.1193 ± 0.0014 | 0.1188 ± 0.0010 |
| 100 _{θмс} | 1.04085 ± 0.00047 | 1.04103 ± 0.00046 | 1.04106 ± 0.00041 | 1.04077 ± 0.00032 | 1.04087 ± 0.00032 | 1.04093 ± 0.00030 |
| τ | 0.078 ± 0.019 | 0.066 ± 0.016 | 0.067 ± 0.013 | 0.079 ± 0.017 | 0.063 ± 0.014 | 0.066 ± 0.012 |
| $\ln(10^{10}A_{\rm s})$ | 3.089 ± 0.036 | 3.062 ± 0.029 | 3.064 ± 0.024 | 3.094 ± 0.034 | 3.059 ± 0.025 | 3.064 ± 0.023 |
| <i>n</i> _s | 0.9655 ± 0.0062 | 0.9677 ± 0.0060 | 0.9681 ± 0.0044 | 0.9645 ± 0.0049 | 0.9653 ± 0.0048 | 0.9667 ± 0.0040 |
| H_0 | 67.31 ± 0.96 | 67.81 ± 0.92 | 67.90 ± 0.55 | 67.27 ± 0.66 | 67.51 ± 0.64 | 67.74 ± 0.46 |
| Ω _Λ | 0.685 ± 0.013 | 0.692 ± 0.012 | 0.6935 ± 0.0072 | 0.6844 ± 0.0091 | 0.6879 ± 0.0087 | 0.6911 ± 0.0062 |
| Ω_m | 0.315 ± 0.013 | 0.308 ± 0.012 | 0.3065 ± 0.0072 | 0.3156 ± 0.0091 | 0.3121 ± 0.0087 | 0.3089 ± 0.0062 |
| $\Omega_{\rm m}h^2$ | 0.1426 ± 0.0020 | 0.1415 ± 0.0019 | 0.1413 ± 0.0011 | 0.1427 ± 0.0014 | 0.1422 ± 0.0013 | 0.14170 ± 0.00097 |
| $\Omega_{\rm m}h^3$ | 0.09597 ± 0.00045 | 0.09591 ± 0.00045 | 0.09593 ± 0.00045 | 0.09601 ± 0.00029 | 0.09596 ± 0.00030 | 0.09598 ± 0.00029 |
| σ_8 | 0.829 ± 0.014 | 0.8149 ± 0.0093 | 0.8154 ± 0.0090 | 0.831 ± 0.013 | 0.8150 ± 0.0087 | 0.8159 ± 0.0086 |

Table 5. Constraints on 1-parameter extensions to the base Λ CDM model for combinations of *Planck* power spectra, *Planck* lensing, and external data (BAO+JLA+H₀, denoted "ext"). Note that we quote 95 % limits here.

| Parameter | TT | TT+lensing | TT+lensing+ext | TT, TE, EE | TT, TE, EE+lensing | TT, TE, EE+lensing+ext |
|-------------------------|----------------------------|----------------------------|-------------------------------|----------------------------|----------------------------|----------------------------|
| Ω _κ | $-0.052^{+0.049}_{-0.055}$ | $-0.005^{+0.016}_{-0.017}$ | $-0.0001^{+0.0054}_{-0.0052}$ | $-0.040^{+0.038}_{-0.041}$ | $-0.004^{+0.015}_{-0.015}$ | 0.0008+0.0040 |
| Σm_{ν} [eV] | < 0.715 | < 0.675 | < 0.234 | < 0.492 | < 0.589 | < 0.194 |
| <i>N</i> _{eff} | $3.13^{+0.64}_{-0.63}$ | $3.13^{+0.62}_{-0.61}$ | $3.15_{-0.40}^{+0.41}$ | $2.99^{+0.41}_{-0.39}$ | $2.94^{+0.38}_{-0.38}$ | $3.04^{+0.33}_{-0.33}$ |
| <i>Y</i> _P | $0.252^{+0.041}_{-0.042}$ | $0.251^{+0.040}_{-0.039}$ | $0.251^{+0.035}_{-0.036}$ | $0.250^{+0.026}_{-0.027}$ | $0.247^{+0.026}_{-0.027}$ | $0.249^{+0.025}_{-0.026}$ |
| $dn_s/d\ln k$ | $-0.008^{+0.016}_{-0.016}$ | $-0.003^{+0.015}_{-0.015}$ | $-0.003^{+0.015}_{-0.014}$ | $-0.006^{+0.014}_{-0.014}$ | $-0.002^{+0.013}_{-0.013}$ | $-0.002^{+0.013}_{-0.013}$ |
| r _{0.002} | < 0.103 | < 0.114 | < 0.114 | < 0.0987 | < 0.112 | < 0.113 |
| w | $-1.54^{+0.62}_{-0.50}$ | $-1.41^{+0.64}_{-0.56}$ | $-1.006^{+0.085}_{-0.091}$ | $-1.55^{+0.58}_{-0.48}$ | $-1.42^{+0.62}_{-0.56}$ | $-1.019^{+0.075}_{-0.080}$ |

Table 4. Parameter 68 % confidence limits for the base Λ CDM model from *Planck* CMB power spectra, in combination with lensing reconstruction ("lensing") and external data ("ext," BAO+JLA+ H_0). Nuisance parameters are not listed for brevity (they can be found in the *Planck Legacy Archive* tables), but the last three parameters give a summary measure of the total foreground amplitude (in μ K²) at ℓ = 2000 for the three high- ℓ temperature spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN (posterior mean $Y_P \approx 0.2453$, with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on $\Omega_b h^2$).

| Parameter | TT+lowP 68 % V.mts | TT+lowP+lensing 68 % limits | TT+lowP+lensing+ext 68 % limits | TT,TE,EE+lowP 68 % limits | TT,TE,EE+lowP+lensing 68 % limits | TT,TE,EE+lowP+lensing+ext 68 % limits |
|-------------------------|-----------------------|--------------------------------|------------------------------------|------------------------------|--------------------------------------|------------------------------------------|
| $\Omega_{\rm b}h^2$ | 0.0222 ± 0.0002. | 0.02226 ± 0.00023 | 0.02227 ± 0.00020 | 0.02225 ± 0.00016 | 0.02226 ± 0.00016 | 0.02230 ± 0.00014 |
| $\Omega_{\rm c}h^2$ | 0.1117 ± 0.0022 | 0.1186 ± 0.0020 | 0.1184 ± 0.0012 | 0.1198 ± 0.0015 | 0.1193 ± 0.0014 | 0.1188 ± 0.0010 |
| 100θ _{MC} | 1.040 5 ± 0.00047 | 1.04103 ± 0.00046 | 1.04106 ± 0.00041 | 1.04077 ± 0.00032 | 1.04087 ± 0.00032 | 1.04093 ± 0.00030 |
| τ | 0.013 ± 0.019 | 0.066 ± 0.016 | 0.067 ± 0.013 | 0.079 ± 0.017 | 0.063 ± 0.014 | 0.066 ± 0.012 |
| $\ln(10^{10}A_{\rm s})$ | 3.08. ± 0.036 | 3.062 ± 0.029 | 3.064 ± 0.024 | 3.094 ± 0.034 | 3.059 ± 0.025 | 3.064 ± 0.023 |
| <i>n</i> _s | 0.9655 ± 0.02 2 | 0.9677 ± 0.0060 | 0.9681 ± 0.0044 | 0.9645 ± 0.0049 | 0.9653 ± 0.0048 | 0.9667 ± 0.0040 |
| H_0 | 67.31 ± 0.96 | 67.81 ± 0.92 | 67.90 ± 0.55 | 67.27 ± 0.66 | 67.51 ± 0.64 | 67.74 ± 0.46 |
| Ω _Λ | 0.685 ± 0.013 | 0.692 ± 0.012 | 0.6935 ± 0.0072 | 0.6844 ± 0.0091 | 0.6879 ± 0.0087 | 0.6911 ± 0.0062 |
| Ω_m | 0.315 ± 0.013 | 0.308 ± 0.012 | 0.3065 ± 0.0072 | 0.3156 ± 0.0091 | 0.3121 ± 0.0087 | 0.3089 ± 0.0062 |
| $\Omega_{\rm m}h^2$ | 0.1426 ± 0.0020 | 0.1415 ± 0.0019 | 0.1413 ± 0.0011 | 0.1427 ± 0.0014 | 0.1422 ± 0.0013 | 0.14170 ± 0.00097 |
| $\Omega_{\rm m}h^3$ | 0.09597 ± 0.00045 | 0.09591 ± 0.00045 | 0.09593 ± 0.00045 | 0.09601 ± 0.00029 | 0.09596 ± 0.00030 | 0.09598 ± 0.00029 |
| σ_8 | 0.829 ± 0.014 | 0.8149 ± 0.0093 | 0.8154 ± 0.0090 | 0.831 ± 0.013 | 0.8150 ± 0.0087 | 0.8159 ± 0.0086 |

Table 5. Constraints on 1-parameter extensions to the base Λ CDM model for combinations of *Planck* power spectra, *Planck* lensing, and external data (BAO+JLA+H₀, denoted "ext"). Note that we quote 95 % limits here.

| Parameter | TT | TT+lensing | TT+lensing+ext | TT, TE, EE | TT, TE, EE+lensing | TT, TE, EE+lensing+ext |
|-------------------------|----------------------------|----------------------------|-------------------------------|----------------------------|----------------------------|----------------------------|
| Ω _κ | $-0.052^{+0.049}_{-0.055}$ | $-0.005^{+0.016}_{-0.017}$ | $-0.0001^{+0.0054}_{-0.0052}$ | $-0.040^{+0.038}_{-0.041}$ | $-0.004^{+0.015}_{-0.015}$ | 0.0008+0.0040 |
| $\Sigma m_{\rm v}$ [eV] | < 0.715 | < 0.675 | < 0.234 | < 0.492 | < 0.589 | < 0.194 |
| <i>N</i> _{eff} | $3.13^{+0.64}_{-0.63}$ | $3.13_{-0.61}^{+0.62}$ | $3.15_{-0.40}^{+0.41}$ | $2.99^{+0.41}_{-0.39}$ | $2.94^{+0.38}_{-0.38}$ | $3.04_{-0.33}^{+0.33}$ |
| <i>Y</i> _P | 0.252+0.041 | 0.251+0.040 | $0.251^{+0.035}_{-0.036}$ | $0.250^{+0.026}_{-0.027}$ | $0.247_{-0.027}^{+0.026}$ | $0.249^{+0.025}_{-0.026}$ |
| $dn_s/d\ln k$ | $-0.008^{+0.016}_{-0.016}$ | $-0.003^{+0.015}_{-0.015}$ | $-0.003^{+0.015}_{-0.014}$ | $-0.006^{+0.014}_{-0.014}$ | $-0.002^{+0.013}_{-0.013}$ | $-0.002^{+0.013}_{-0.013}$ |
| r _{0.002} | 0.102 | < 0.114 | < 0.114 | < 0.0987 | < 0.112 | 0.112 |
| w | $-1.54^{+0.62}_{-0.50}$ | $-1.41^{+0.64}_{-0.56}$ | $-1.006^{+0.085}_{-0.091}$ | $-1.55^{+0.58}_{-0.48}$ | $-1.42^{+0.62}_{-0.56}$ | $-1.019_{-0.080}^{+0.075}$ |

Take the hydrostatic mass estimates for M - T

Take the hydrostatic mass estimates for M - T

Add an offset to the mass : $M_{HE} = (1 - b)M_{true}$ with 1 - b = 0.8

Take the hydrostatic mass estimates for M - T

Add an offset to the mass : $M_{HE} = (1 - b)M_{true}$ with 1 - b = 0.8

Compute the number of clusters expected in the Λ CDM model with the Planck selection function.

Planck SZ counts

Planck SZ counts



3 possible solutions...

3 possible solutions...

The CMB produced biased results...

- 3 possible solutions...
- The CMB produced biased results...
- Clusters are not selected exactly as expected (selection function issue).

3 possible solutions...

The CMB produced biased results...

Clusters are not selected exactly as expected (selection function issue).

This is the indication of new physics...