cosmologie et énergie noire

Martin Kunz
Université de Genève

(& the Euclid & Planck collaborations & some others)

???

Who are you?

A) I have not started my PhD yet

B) first year of PhD

C) second year of PhD

D) 3+ years

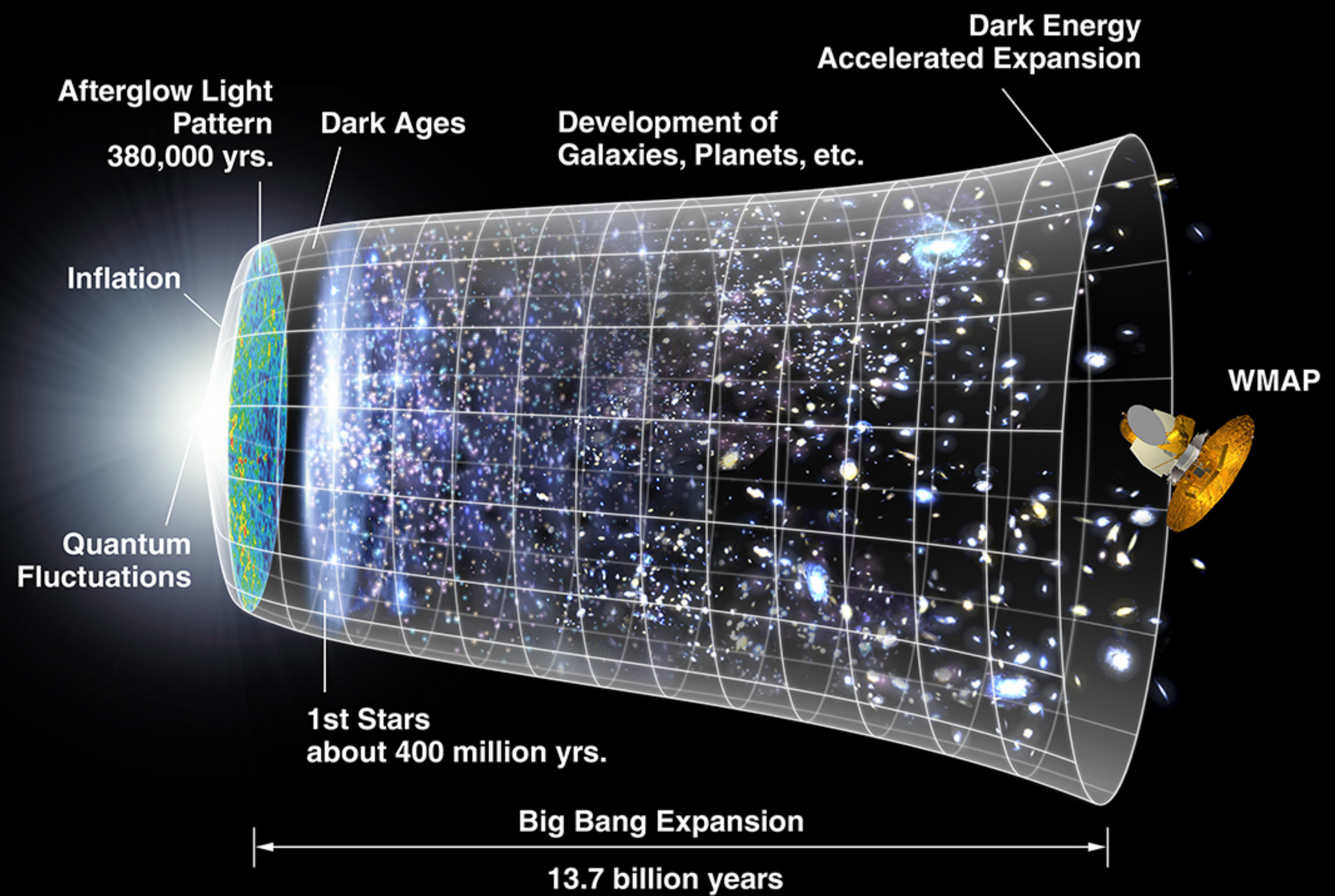
E) Engineer

F) post-doc, staff, …

global outline

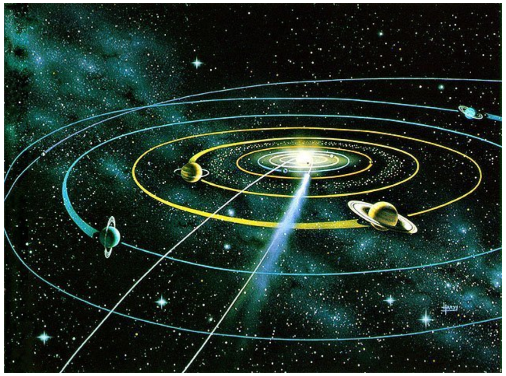
- **background: the FLRW metric**
  - metric, scale factor, redshift, distances
  - Einstein eqn's, evolution of the universe
  - the cosmological standard model: LCDM
- **the perturbed universe**
  - inflation, evolution of the perturbations
  - brief discussion of CMB
- **dark energy & modified gravity**
  - dark energy models
  - screening
  - effective field theory
  - phenomenological modeling
  - Planck constraints, expectations for future constraints

Brief history of the Universe



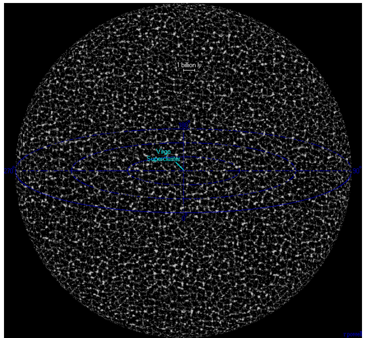
orders of magnitude

cosmology also goes right down to the Planck scale…
… but for now we are more interested in large scales!



solar system:
size: billions of km (10^9 km)
1AU = 1.5×10^8 km
Pluto ~ 40 AU, Voyager 1: 128 AU

galaxies:
size ~ 10 kpc
1pc ≈ 3 light years = 3×10^{13} km
billions of stars (sizes vary!)



(observable) universe
size ~ 10 Gpc (~ 10^{23} km vs $l_P \sim 10^{-38}$ km)
~ 10^{11} galaxies

Outline of part I
how to describe the universe

- **metric structure: cosmography**
  - the metric
  - expansion of the universe, redshift and Hubble's law
  - cosmological distances and the age of the universe
- **content and evolution of the universe**
  - Einstein equations and the Bianchi identity
  - the critical density and the Ω's
  - the evolution of the universe
  - contents, the LCDM model

???

- Friedmann equation
  - A) I know it very well
  - B) I have seen it
  - C) what? never heard of it

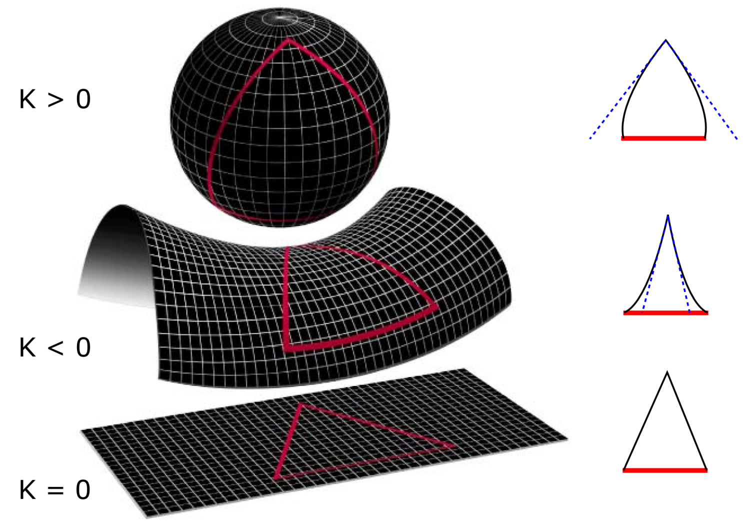
- Age of the Universe
  - A) I don’t know it, and I don’t know how to compute it
  - B) I know it, but can’t compute it
  - C) I can compute it, but have forgotten the value
  - D) Come on, do you think I’m stupid or what?

the cosmological space-time

Ingredients:

- the universe looks isotropic around us
- **Cosmological principle**: all observers are equivalent
- some technical assumptions on how stuff behaves
- implication: the universe has a **FLRW metric**

$$ds^2 = dt^2 - \left(\frac{dR^2}{1-KR^2} + R^2 d\Omega\right)$$



(at least for simply connected spaces)

basic quantities

- Maximal symmetry for spatial sections imposes an even stronger constraint: setting R(t) = a(t) r, the line element has the form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega \right)$$

where k = ±1 or 0 is a constant

- For this metric, the curves (r,θ,ϕ)=const are geodesics for a 4-velocity u=(1,0,0,0) since $\Gamma^{\mu}{}_{00}$=0 [check!] -> **comoving coordinates**

(geodesic eqn: $\ddot{X}^{\mu} + \Gamma^{\mu}_{\alpha\beta}\dot{X}^{\alpha}\dot{X}^{\beta} = 0$)

- expansion leads to redshift

$$1 + z = \frac{a(t_0)}{a(t_1)}$$

The Hubble law

for two galaxies at a fixed **comoving** distance r_0:

physical distance $x(t) = a(t)r_0$

-> apparent motion: $$\frac{dx}{dt} = \dot{a}r_0 = \frac{\dot{a}}{a}x \equiv H_0 x$$

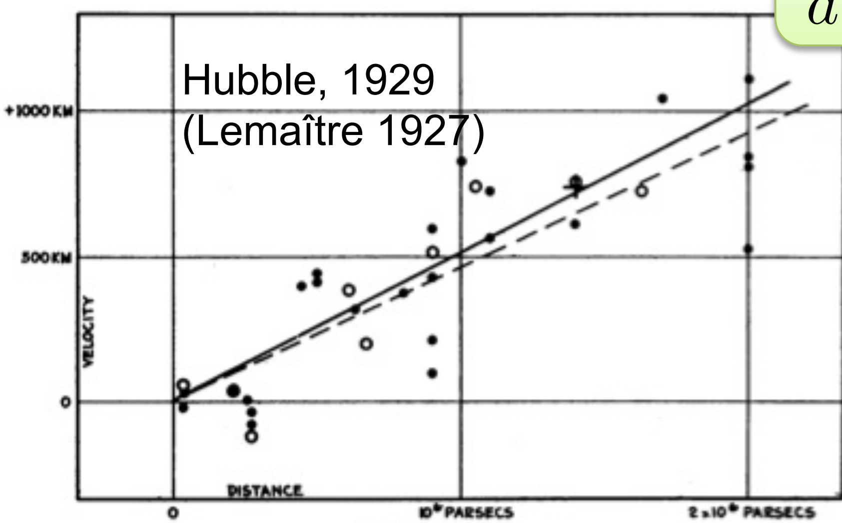
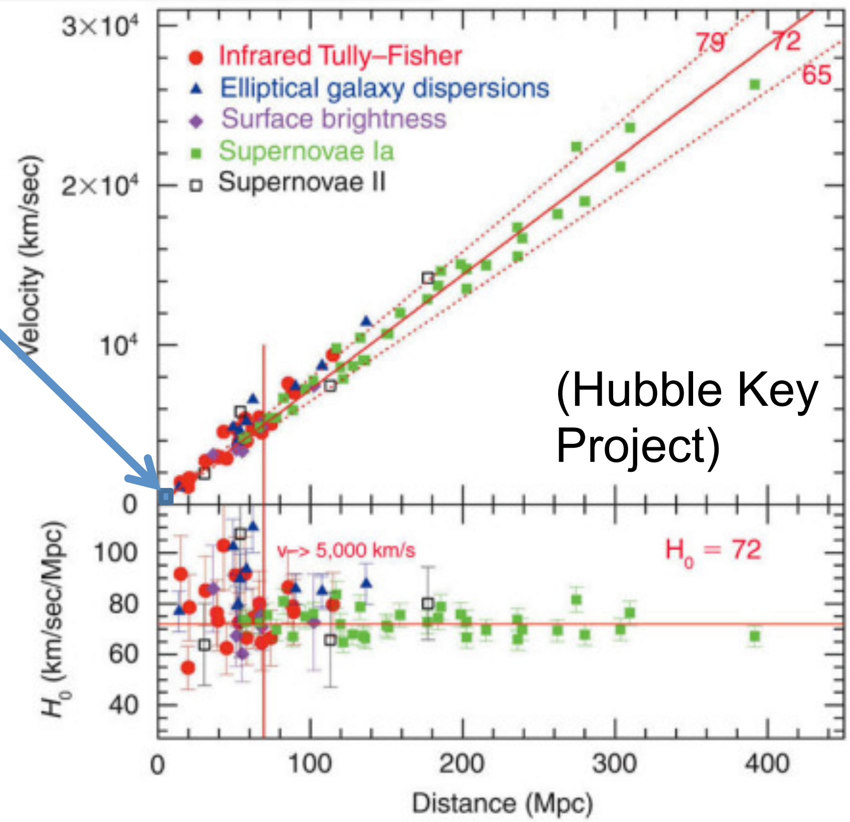


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.



philosophical remarks

- The FLRW metric is just picked ‘by hand’

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega \right)$$

- ***This needs to be tested as much as possible!***
- E.g. an even more symmetric possibility would be the de Sitter metric, but observations rule it out!
- We know that the Universe is not exactly FLRW, it’s not entirely clear yet how important this is
- FLRW leads to testable consequences (the ‘3 pillars’ – there are more tests)
- Unfortunately we have only 1 Universe, and we can’t even go everywhere, we can only observe

cosmological distances

simpler to transform the distance variable r to χ:

$$r = S_\kappa(\chi) = \begin{cases} \sin\chi & \kappa = +1 \\ \chi & \kappa = 0 \\ \sinh\chi & \kappa = -1 \end{cases}$$

$$\Rightarrow ds^2 = dt^2 - a^2(t)\left(d\chi^2 + S_\kappa(\chi)^2 d\Omega\right)$$

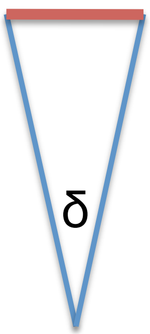
$$\Rightarrow dV = a_0^2 S_\kappa(\chi)^2 d\Omega d\chi$$ volume element today

we can now *define* a «metric» distance:

$$\boxed{d_m(\chi) = a_0 S_\kappa(\chi)} \qquad \chi = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{a_1}^{a_0} \frac{da}{a\dot{a}} = \frac{1}{a_0}\int_0^{z_1} \frac{dz}{H(z)}$$

cosmological distances

but physical distances need to be **observables**!



1) angular diameter distance: object of physical size D observed under angle δ, but photons were emitted at time $t_1 < t_0$:

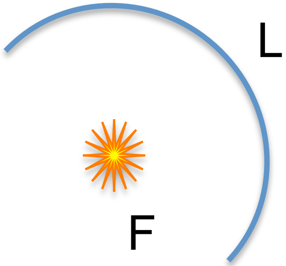
$$D = a(t_1) S_\kappa(\chi)\delta = \frac{a(t_1)}{a_0} a_0 S_\kappa(\chi)\delta \equiv d_A \delta$$

$$\boxed{d_A = \frac{1}{1+z} d_m}$$

2) luminosity distance: consider observed flux F for an object with known intrinsic luminosity L («standard candle»)

$$F \equiv \frac{L}{4\pi d_L^2}$$

surface: $4\pi d_m^2$

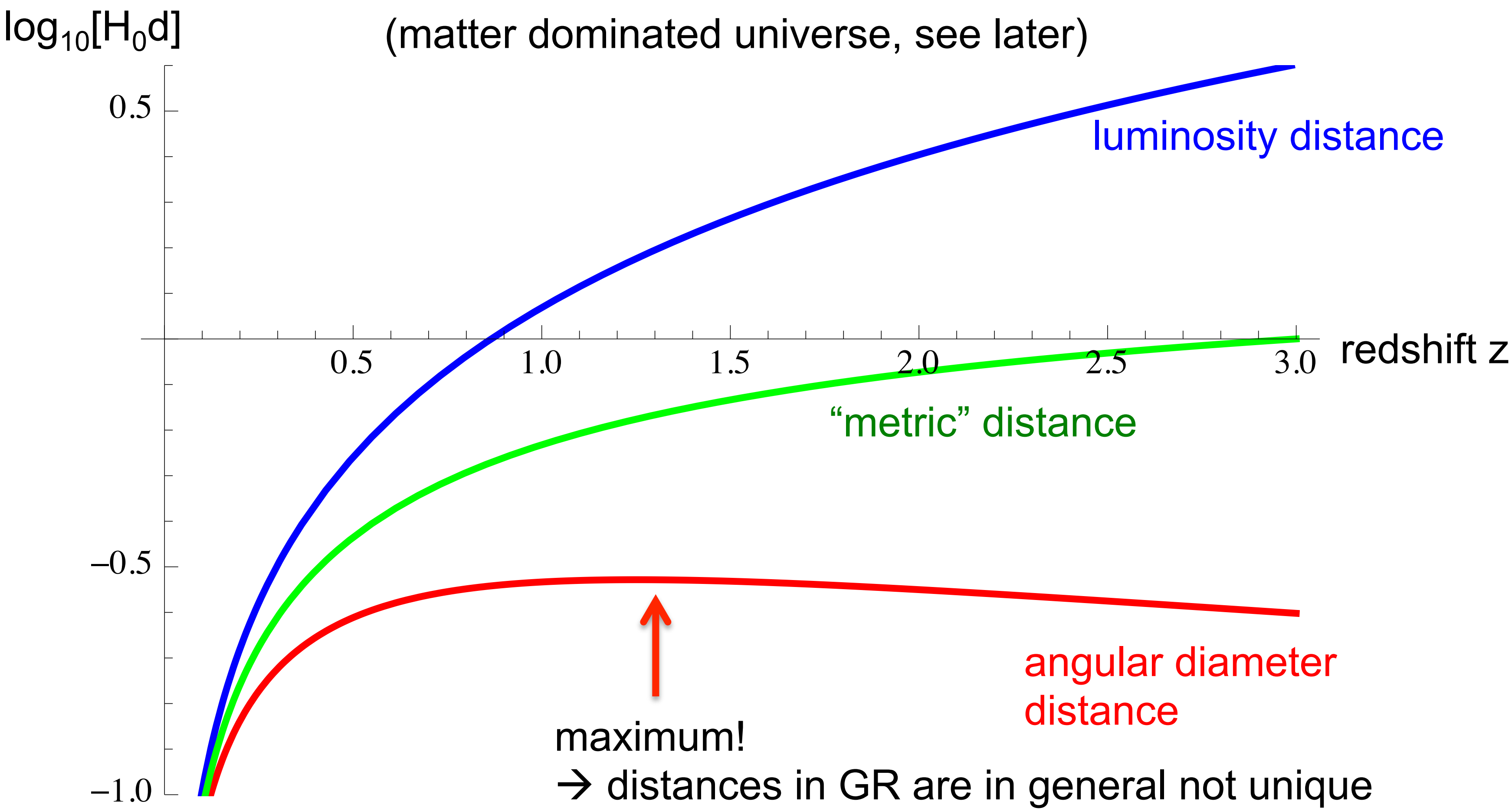


source emitting one photon per second:

1) redshift
2) increased time between arrivals

$$\boxed{d_L = (1+z) d_m}$$

distance example



remark: $d_L = (1+z)^2 \, d_A$ is very general

age of the universe

computing the age of the universe is very straightforward:

$$t_0 = \int_0^{t_0} dt = \int_0^{a_0} \frac{da}{\dot{a}} = \int_0^{a_0} \frac{da}{aH(a)} = \int_0^{\infty} \frac{dz}{H(z)(1+z)}$$

but we need to know the evolution of the scale factor a(t). This in turn depends on the contents of the universe…

cue Einstein: $\underbrace{G_{\mu\nu}}_{\text{geometry } [=f(g_{\mu\nu})]} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{content}}$

???

So far, the lecture is

A) too fast

B) too slow

C) both

D) neither

E) what’s the point?!

what is in the universe?

- homogeneous and isotropic metric: matter does also have to be distributed in this way
- in **some** coordinate system the energy momentum tensor has the form:

$$T^i_0 = 0, \quad T^1_1 = T^2_2 = T^3_3$$

and the components depend only on time

$$\boxed{T^\nu_\mu = \mathrm{diag}\left(\rho(t), -p(t), -p(t), -p(t)\right)}$$

- the pressure determines the nature of the fluid, p = w ρ:
  - w = 0 : pressureless 'dust', 'matter'
  - w = 1/3 : radiation
  - what is w for $T_{\mu\nu} = \Lambda g_{\mu\nu}$?

the conservation equation

- Bianchi identity (geometric identity for $G_{\mu\nu}$):

$$T^{\mu\nu}_{;\mu} = 0 = G^{\mu\nu}_{;\mu}$$

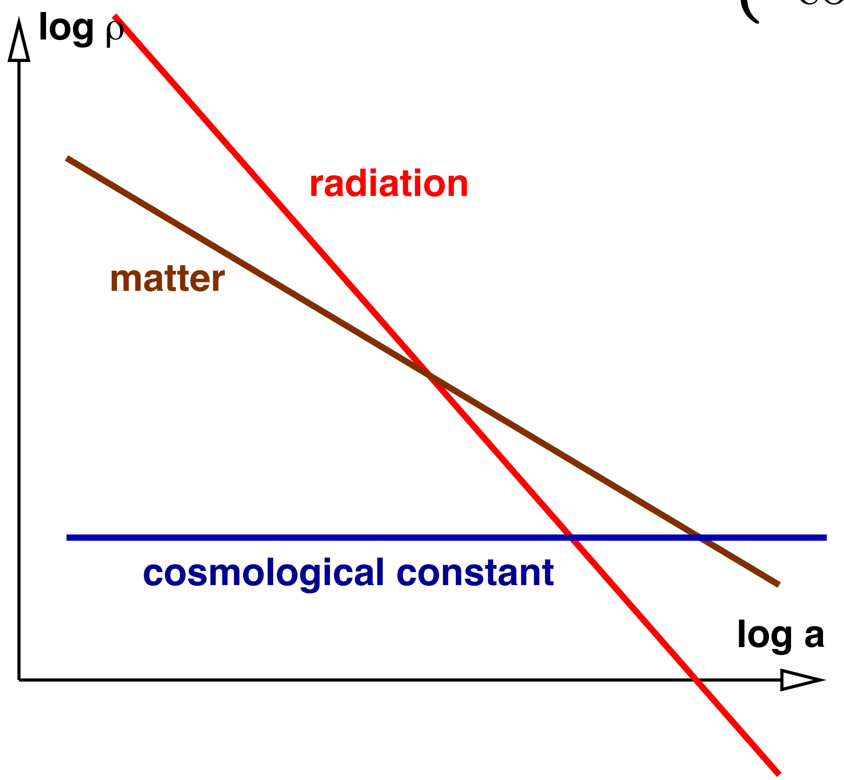
$$T^{\nu}_{0;\nu} = \dot{\rho} + \Gamma^{i}_{i0}(\rho + p) = \boxed{\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\underbrace{(\rho + p)}_{(1+w)\rho} = 0}$$

Questions (3 minutes, in groups):

- for a constant w, what is the evolution of ρ(a)?
  (eliminate the variable t from the equation)
- for the three cases w = 0, 1/3, -1, what is ρ(a)?
- does the result make sense?

evolution of the energy densities

$$\rho \propto a(t)^{-3(1+w)} \propto \begin{cases} a(t)^{-3} & \text{for } w = 0 \quad \text{(matter)} \\ a(t)^{-4} & \text{for } w = 1/3 \quad \text{(radiation)} \\ \text{const.} & \text{for } w = -1 \quad \text{(vacuum energy)} \end{cases}$$



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A) I knew the right answer already
B) I found the right answer
C) I made a mistake, but know why
D) I don't know how to get the answer

Einstein equations

- we now have all necessary ingredients to compute the **Einstein equations**:
  - **metric**
  - **energy-momentum tensor**

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu} \qquad R \equiv g^{\mu\nu} R_{\mu\nu}$$

$$R^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\nu\beta,\mu} - \Gamma^{\alpha}_{\mu\beta,\nu} + \Gamma^{\delta}_{\nu\beta}\Gamma^{\alpha}_{\mu\delta} - \Gamma^{\delta}_{\mu\beta}\Gamma^{\alpha}_{\nu\delta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right)$$

try to do it yourselves… ☺

Friedmann equations

you should find:

$$R_{00} = -3\frac{\ddot{a}}{a} \qquad R_{ij} = -\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\kappa}{a^2}\right] g_{ij}$$

$$R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2}\right]$$

the **space-time** curvature is non-zero even for k=0!

0-0 component:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

sum of ρ from all types of energy

i-i component:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = -8\pi G_N p$$

Friedmann equations II

three comments:

- you can combine the two equations to find

  $$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_N}{3}\left(\rho + 3p\right)$$

  -> the expansion is accelerating if $p<-\rho/3$

- the two Einstein equations and the conservation equation are not independent
- there are 3 unknown quantities (ρ, p and a) but only two equations, so one quantity needs to be given (normally p) – as well as the constant k.

the critical density

Friedmann eq. $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

$$H \equiv \left(\frac{\dot{a}}{a}\right) \qquad \frac{\kappa}{a^2 H^2} = \frac{8\pi G_N \rho}{3H^2} - 1 \equiv \frac{\rho}{\rho_c} - 1 \equiv \Omega - 1$$

$$\Omega(t) > 1 \quad \Rightarrow \quad \kappa > 0 \Rightarrow \textbf{closed} \text{ universe}$$

$$\Omega(t) = 1 \quad \Rightarrow \quad \kappa = 0 \Rightarrow \textbf{flat} \text{ universe}$$

$$\Omega(t) < 1 \quad \Rightarrow \quad \kappa < 0 \Rightarrow \textbf{open} \text{ universe}$$

and: $$\frac{d}{dt}\underbrace{\left(\frac{\Omega - 1}{\kappa}\right)}_{|\Omega - 1| \quad (\kappa \neq 0)} = \frac{d}{dt}\frac{1}{\dot{a}^2} = -2\frac{\ddot{a}}{\dot{a}^3}$$

>0 for expanding universe filled with dust or radiation (and k ≠ 0)
-> the universe becomes “less flat”
-> strange (why?)

‘Ω form’ of Friedmann eq.

Friedmann eq. $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

evolution of ρ for the «usual» 4 constituents:

- radiation: a^{-4}
- dust: a^{-3}
- curvature: a^{-2} ($H^2 + k/a^2 \sim \rho$)
- cosmological constant: a^0

notation:

$$\Omega_X = \left.\frac{\rho_X}{\rho_c}\right|_{t_0}$$

$$H^2 = H_0^2 \left[\frac{8\pi G}{3H_0^2}\rho_0 \left(\frac{a}{a_0}\right)^{-n} + \ldots + \frac{\kappa}{H_0^2 a_0^2}\left(\frac{a}{a_0}\right)^{-2}\right]$$

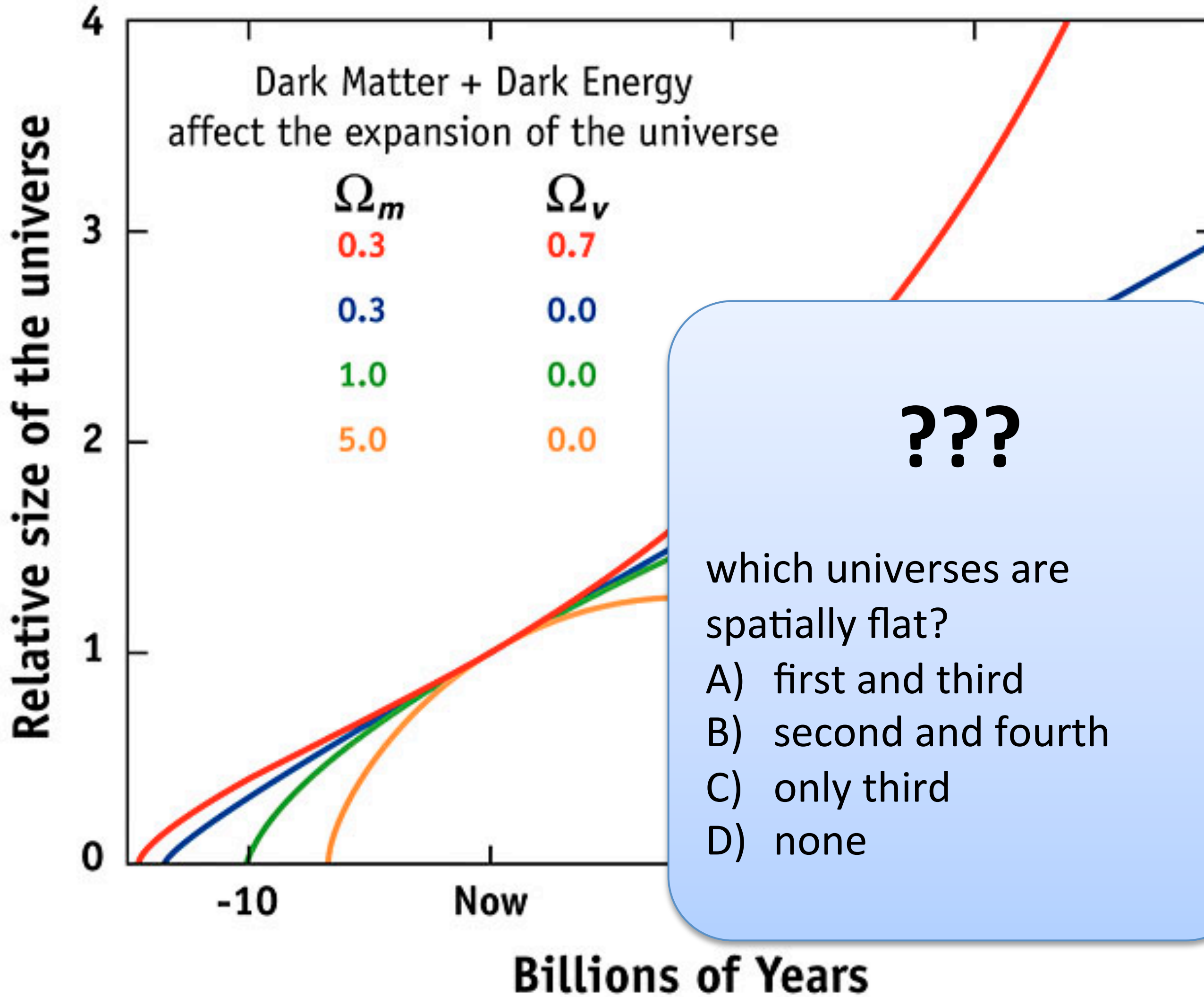
$$H^2 = H_0^2 \left[\Omega_r \left(\frac{a}{a_0}\right)^{-4} + \Omega_m \left(\frac{a}{a_0}\right)^{-3} + \Omega_\Lambda + \Omega_\kappa \left(\frac{a}{a_0}\right)^{-2}\right]$$

$$\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_\kappa = 1$$

evolution of the universe

evolution depends on content!

pure radiation: $a \sim t^{1/2}$
pure matter: $a \sim t^{2/3}$
pure Λ: $a \sim e^{Ht}$



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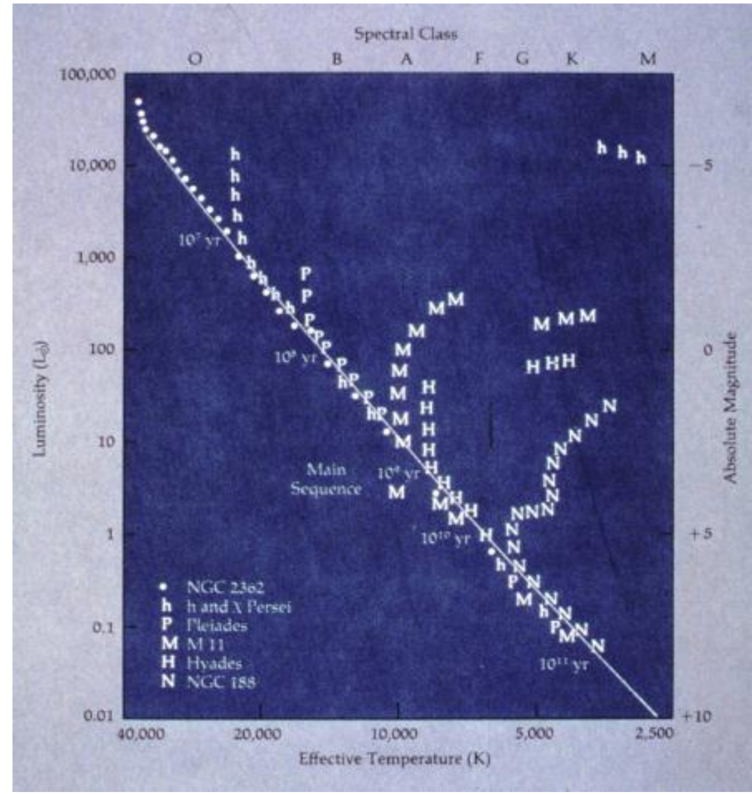
which universes are spatially flat?

A) first and third
B) second and fourth
C) only third
D) none

age of the universe revisited

we had: $$t_0 = \int_0^\infty \frac{dz}{H(z)(1+z)}$$



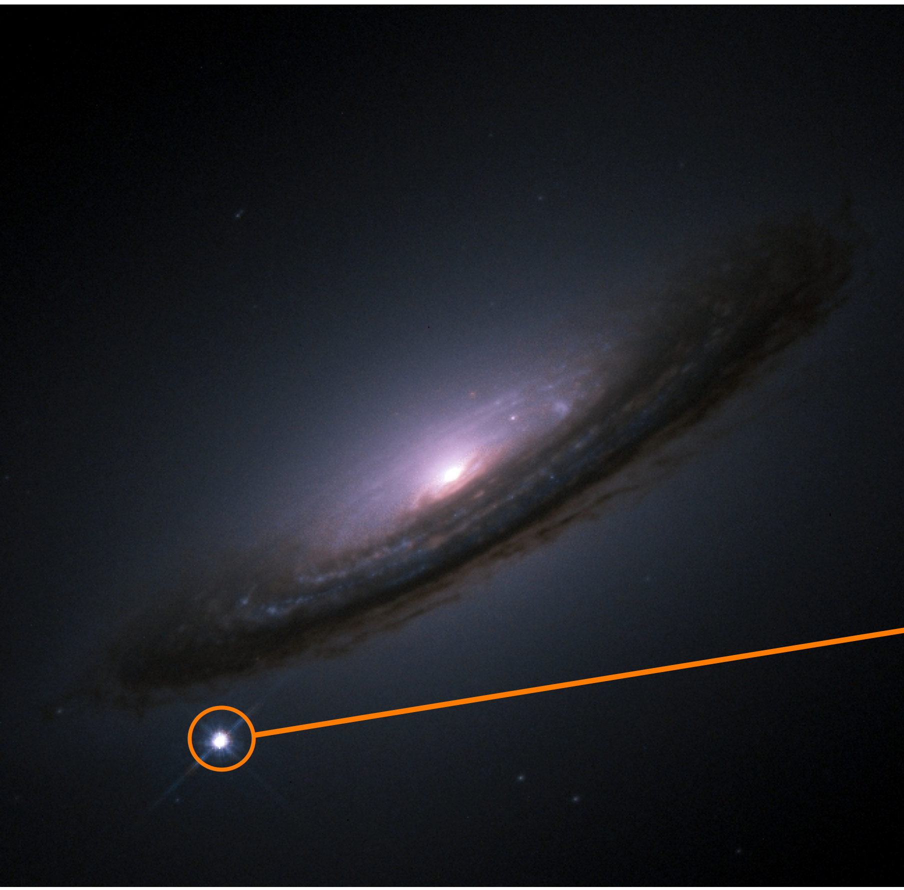
but for a matter-dominated universe:

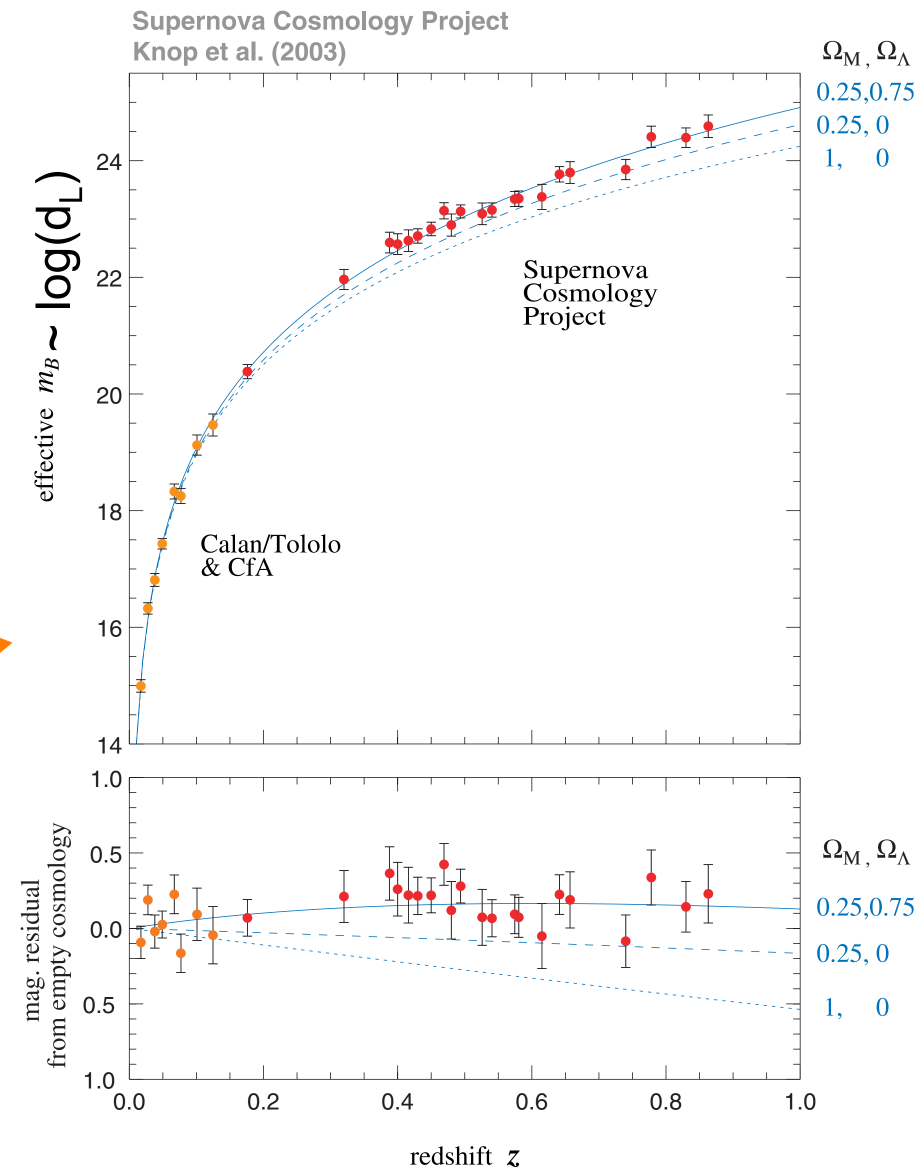
$$H = H_0 \left(\frac{a}{a_0}\right)^{-3/2} = H_0(1+z)^{3/2}$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)^{5/2}} = \int_1^\infty \frac{du}{u^{5/2}} = -\frac{2}{3}\left.\frac{1}{u^{3/2}}\right|_1^\infty = \frac{2}{3}$$

$1/H_0$ ~ 9.8 Gyr/[H_0/100 km/s/Mpc] ~ 13.6 Gyr -> t_0 ~ 9 Gyr but oldest globular star clusters are older: 11-18 Gyr …??!!

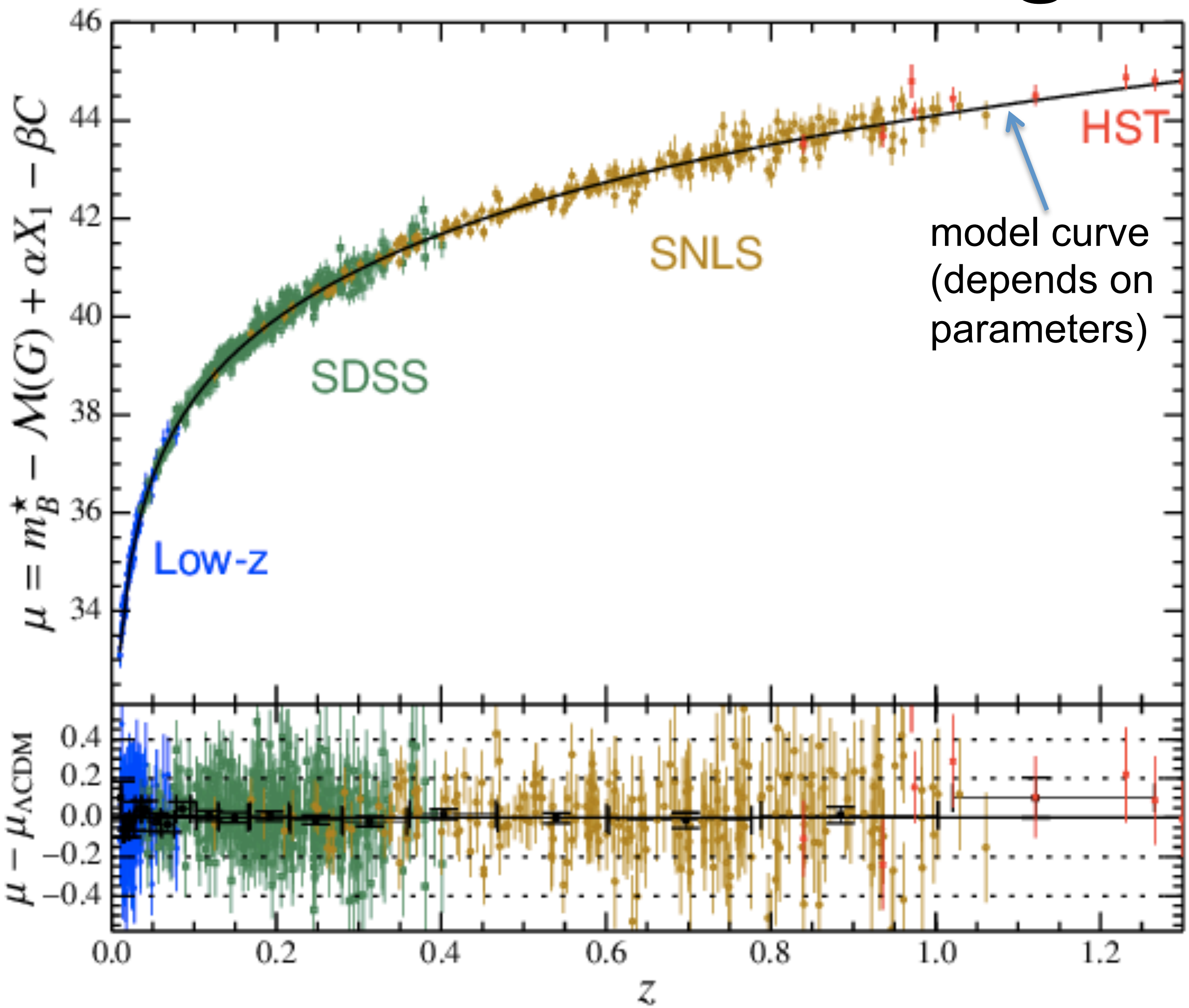
distances revisited





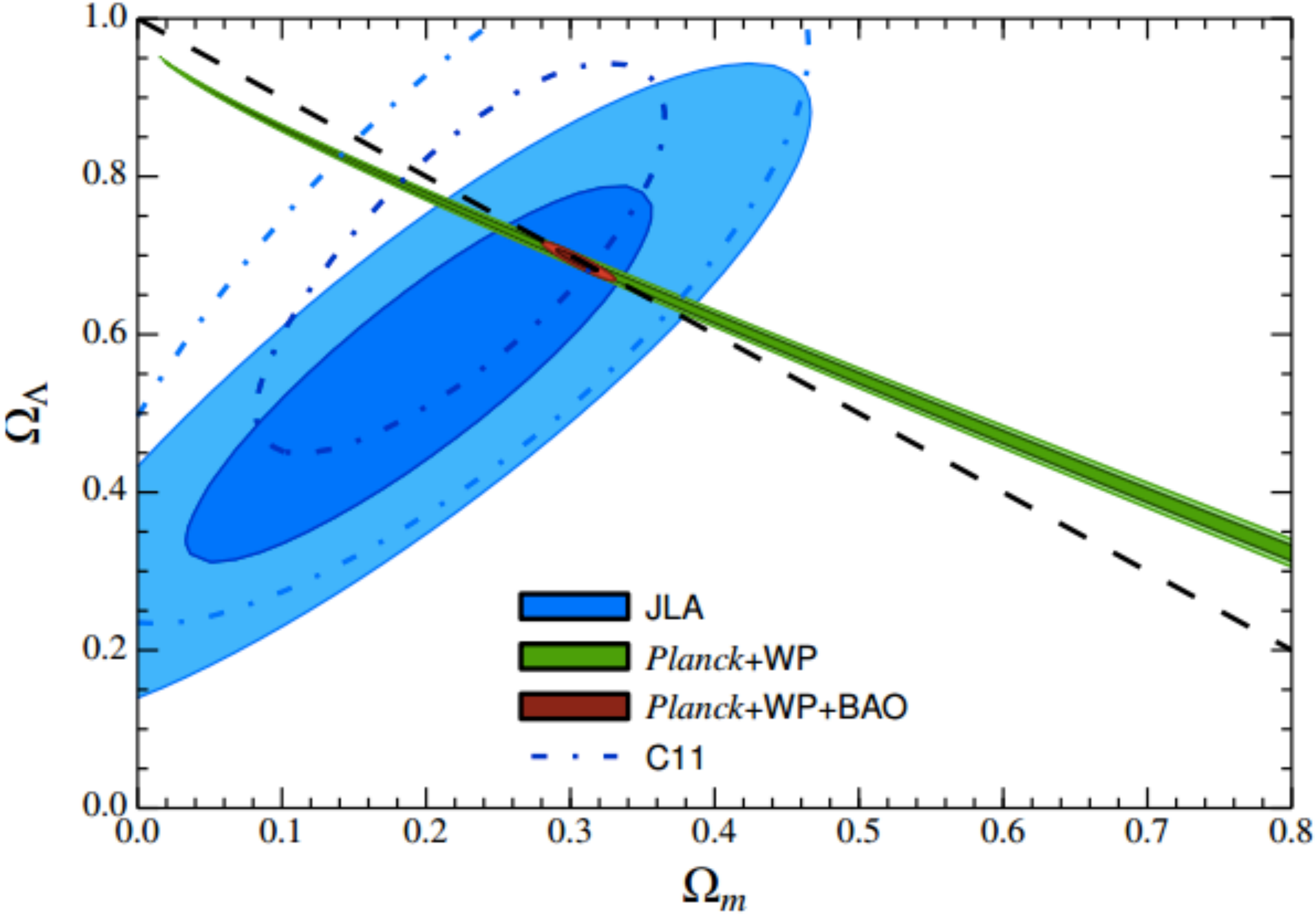
The luminosity distance depends on the contents of the universe through the expansion rate!

current distance diagram



JLA (joint light-curve analysis), SDSS & SNLS

constraints



JLA (joint light-curve analysis), SDSS & SNLS

ingredients for LCDM soup

To explain supernova distances we need:

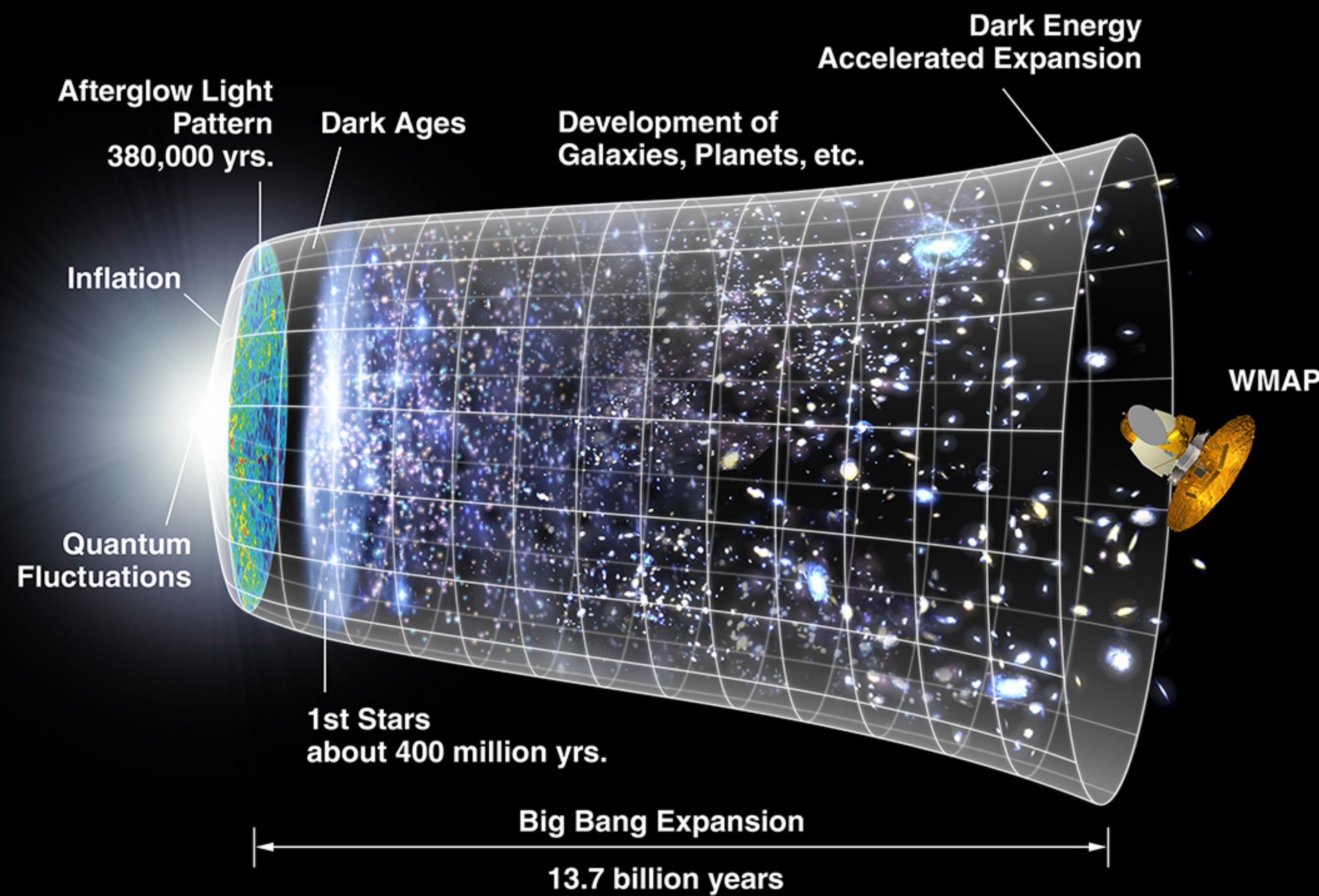
- **(expansion rate: H_0)**
- **(radiation)**
  - given by T_0 through Stefan-Boltzmann
  - includes neutrinos
- **matter: Ω_m**
  - 'normal' and dark
  - "cold" → low velocity and collisionless
- **cosmological constant: Ω_Λ**

→ Lambda-cold-dark-matter model

status report

- **reasonable (?) assumptions → FLRW metric**
- **GR: link of evolution and contents**
  - universe expanding: smaller and hotter in the past
  - age & distance measurements: LCDM model
- **Issues:**
  - universe appears spatially flat
  - where does the structure come from?
  - how do perturbations evolve?
- **Next steps:**
  - inflation with scalar fields
  - creation and evolution of perturbations
  - CMB
  - dark energy / modified gravity

Brief history of the Universe



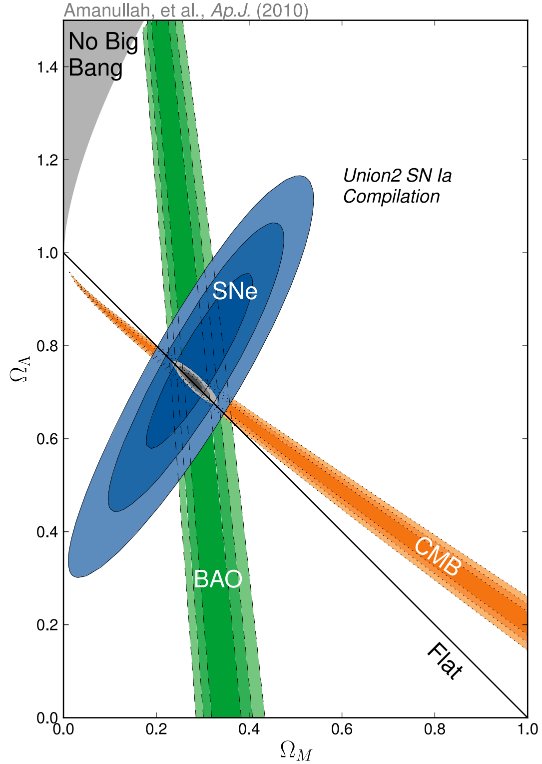
why is the world flat?

we saw:

$$\frac{d}{dt}\underbrace{\left(\frac{\Omega-1}{\kappa}\right)}_{|\Omega-1|\quad(\kappa\neq 0)} = \frac{d}{dt}\frac{1}{\dot{a}^2} = -2\frac{\ddot{a}}{\dot{a}^3}$$

>0 for expanding universe filled with dust or radiation (and k ≠ 0)
-> the universe becomes "less flat"
-> Ω=1 is an unstable fix-point

Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)



following the evolution back in time, we find that (during radiation domination, i.e. before t_{eq})

$$|\Omega(t)-1| \approx 10^{-4}\left(\frac{1\mathrm{eV}}{T}\right)^2$$

BBN: T ≈ 1 MeV -> $|\Omega-1| < 10^{-16}$
Planck: T ≈ 10^{19} GeV -> $|\Omega-1| < 10^{-60}$

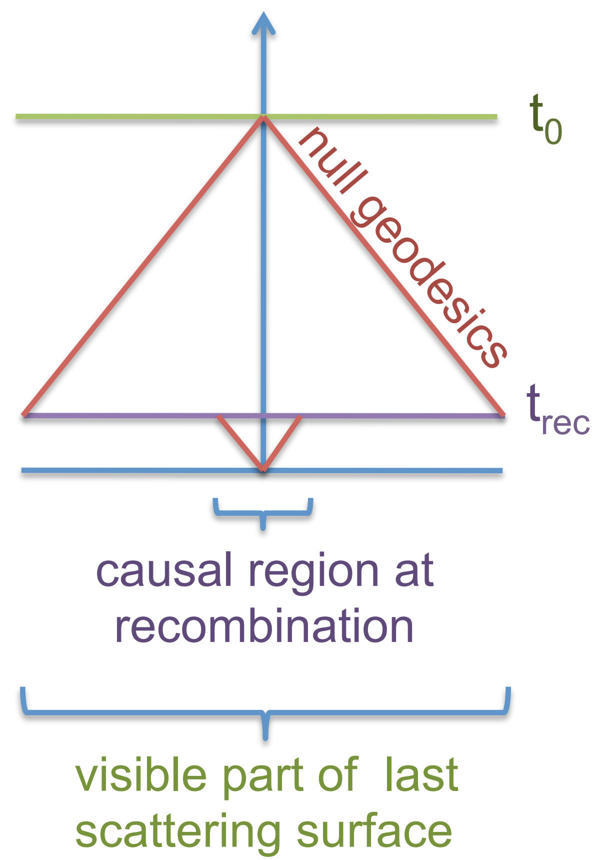
-> what fine-tuned the initial conditions?

why is the sky uniform?

- distance travelled by light: $r = \int \frac{dt}{a(t)}$ (= conformal time)
- distance to last scattering surface: $$r_0 = \int_{t_{\rm rec}}^{t_0} \frac{dt}{a(t)} \approx 3t_0$$
- distance travelled from big bang to recombination: $$r_c = \int_0^{t_{\rm rec}} \frac{dt}{a(t)}$$

in general $r_c << r_0$, unless $a(t) \sim t^\beta$ with $\beta \geq 1 \Leftrightarrow w \leq -1/3$!

since $\boxed{a(t) \propto t^{2/(3+3w)}}$



how to solve the problems

all the problems disappear if $\ddot{a} > 0$ for long enough!

Since $\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_N}{3}(\rho + 3p)$ this needs $p < -\rho/3$

We have seen that for Λ : $p = -\rho$, but forever

-> we need a way to have evolving eq. of state

Solution: use a field … what kind of field? When in doubt, try a scalar field ☺

scalar fields in cosmology

GR + scalar field: $$S = S_g + S_\phi = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right)$$

gravity e.o.m. (Einstein eq.): $$\frac{\delta S[g_{\mu\nu},\phi]}{\delta g^{\mu\nu}} = 0 \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

entries in scalar field EM tensor (FLRW metric)

$$\left\{ \begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \right.$$

scalar field e.o.m. : $$\frac{\delta S[g_{\mu\nu},\phi]}{\delta \phi} = 0 \qquad \ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$

- this is the general method to compute Einstein eq., EM tensor and field e.o.m. from any action
- w=p/ρ for scalar fields can vary, as a function of V(ϕ)

the inflaton eq. of state

$$\left.\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned}\right\}$$

$\dot{\phi}$ small -> p ≈ -ρ, w ≈ -1 (**slow roll**)

$\dot{\phi}$ large -> p ≈ ρ, w ≈ +1

=> slow roll is just what we need

slow-roll approximation: $$3H\dot{\phi} = -V' \qquad H^2 = \frac{1}{3m_P^2}V$$

slow-roll parameters: $$\epsilon(\phi) \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 \quad \eta(\phi) \equiv m_P^2 \frac{V''}{V}$$

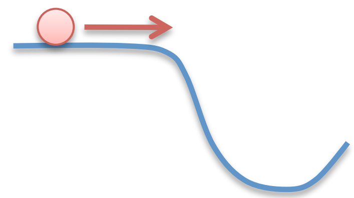
$\epsilon \ll 1\,, \quad |\eta| \ll 1$ for slow-roll -> flat pot.

SR approx => $\dot{\phi}^2 = \frac{2}{3}\epsilon V \Rightarrow p = \left(\frac{2}{3}\epsilon - 1\right)\rho \;\longrightarrow\; \ddot{a} > 0 \leftrightarrow \epsilon < 1$

(first order in ε)

prototypical inflation models

- small field



e.g. $V = V_0\,[1-(\phi/\mu)^{\alpha}]$, $\alpha = 2,4,\ldots$
original inflation: 1st order phase transition -> exit problem

- chaotic / large field

e.g. $V = m^2\phi^2$ or
$V \sim \phi^4$
also eternal inflation models

- hybrid / multifield

- curvaton, N-flation, cyclic models, …

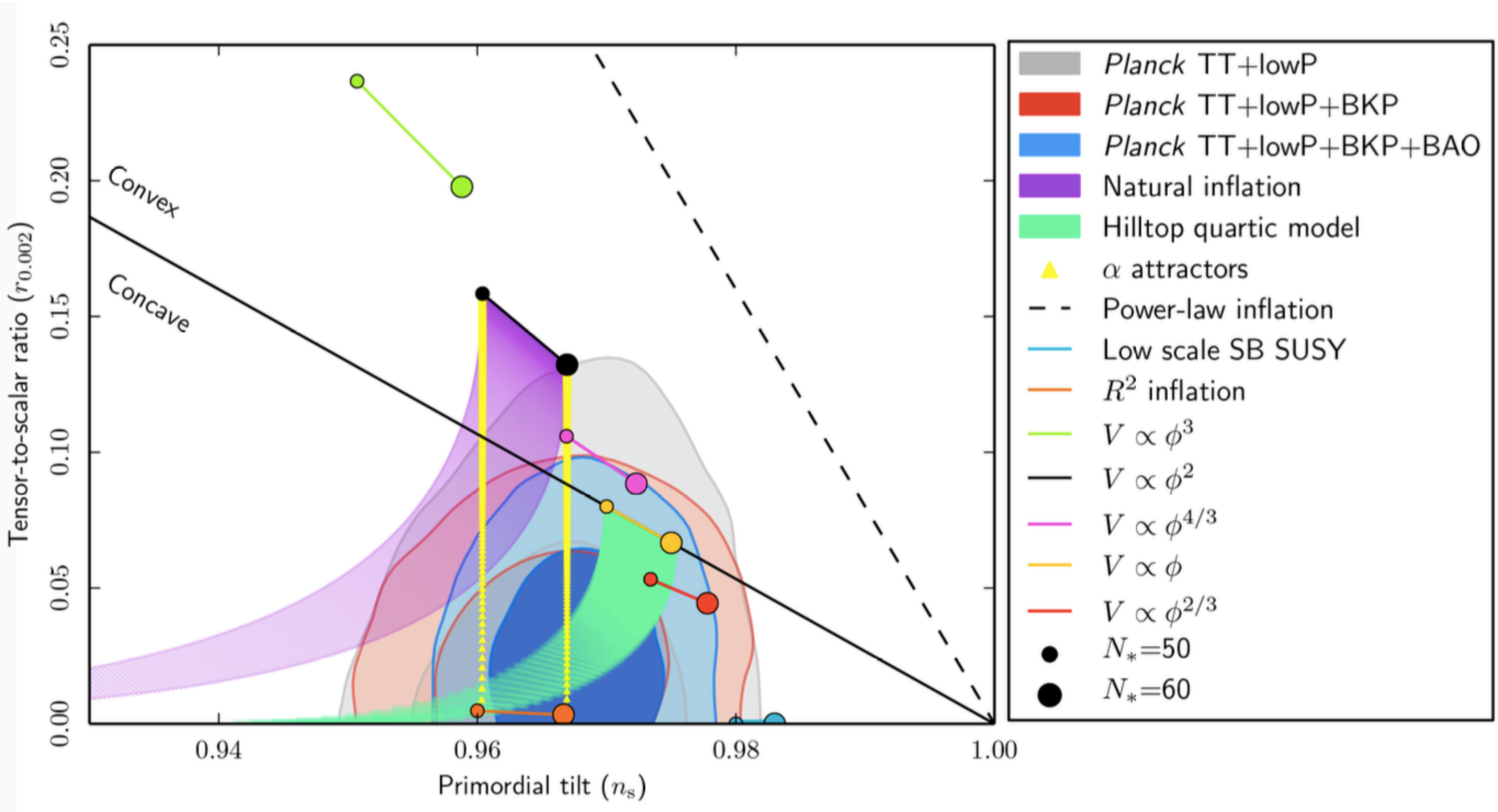
-> large number of inflation scenarios
-> not all work // initial conditions generally problematic

more on inflation

- duration of inflation
  - measured in e-foldings N ~ ln(a)
  - typically 40-60 e-foldings needed to solve the problems we discussed
- at the end of inflation we need to reheat the universe
  - radiation and matter strongly diluted due to expansion
  - energy stored in inflation field dumped into mat/rad during oscillations at bottom of potential
- fluctuations & primordial power spectrum
  - particle creation during inflation
    → are we quantum fluctuations?
  - prediction: nearly Gaussian fluctuations with nearly scale invariant spectrum
- primordial gravitational waves
  - all light d.o.f. acquire fluctuations!

constraints on inflation

As discussed in a bit, the fluctuations visible in the CMB are (believed to be and consistent with) a processed version of the initial fluctuations



generic predictions of inflation

- universe large and nearly flat

  -> **okay**
- nearly (but not quite) scale-invariant spectrum of adiabatic perturbations

  -> **okay** [killed defects]
- (nearly) Gaussian perturbations

  -> **okay** [deviations -> constrain models]
- perturbations on all scales, including super-horizon

  -> **okay** [kills all "causal" sources of perturb.]
- primordial gravitational waves

  -> ??? ("smoking gun" for acc. exp.)

beyond SR inflation

- **single-field slow roll inflation**: nearly scale invariant adiabatic Gaussian perturbations
- more general models: can create
  - **non-Gaussianity**
  - **isocurvature perturbations**
  - **features in the power spectrum**
- realistic (multi-field) models often form **cosmic strings** at the end of inflation
- if detected, such signatures would give important information on fundamental physics of inflation!
- These things can also show in large-scale structure observations!

evolution of the perturbations

- From inflation we have a nearly scale invariant spectrum of perturbations…
  - how will they evolve?
  - what do we observe today?

-> **matter power spectrum / galaxy distribution**

- compute evolution of density perturbations of the dark matter and baryons

-> **CMB power spectrum**

- compute evolution of the perturbations in the radiation

???

- I know perturbation theory
  - A) relatively well
  - B) not in detail, but I have used perturbation equations
  - C) I know what P(k) and C_l are
  - D) not really

- I know what CMB anisotropies are
  - A) I have used Boltzmann codes and Planck likelihoods
  - B) I know what the CMB spectrum shows
  - C) I have heard of the CMB, but I don’t really know
  - D) CMB, what is this?

k-space, power spectra

We tend to use ‘k’-space (Fourier space):

- only perturbations have spatial dependence, so that linear differential eqn’s -> ODE’s in time
- ‘scales’ instead of ‘location’

physical wavelength vs comoving wave number: $\lambda = \frac{2\pi a(t)}{k}$

Fluctuations are random

- need a statistical description -> power spectrum
- power spectra: $P(k) = <|\text{perturbations}(k)|^2>$
- <…> : average over realisations (theory) or over independent directions or volumes (observers)
- Gaussian fluctuations -> P(k) has full information

perturbation theory

basic method:

- set $g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2 h_{\mu\nu}$ $\qquad T_\mu^\nu = \bar{T}_\mu^\nu + \delta T_\mu^\nu$
- stick into Einstein and conservation equations
- linearize resulting equation (order 0 : "background evol.")

⇒ two 4x4 symmetric matrices -> 20 quantities

⇒ we have 4 extra reparametrization d.o.f. -> can eliminate some quantities ("gauge freedom")

⇒ at linear level, perturbations split into "scalars", "vectors" and "tensors", we will mostly consider scalar d.o.f.

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2$$

⇒ do it yourself as an exercise ☺

scalar perturbation equations

Einstein equations:
r.h.s. summed over "stuff" in universe

δ = δρ/ρ density contrast
V divergence of velocity field

$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \left(\delta_i + 3Ha\frac{V_i}{k^2}\right)$$

$$k^2(\phi - \psi) = 12\pi G a^2 \sum_i (1+w_i)\rho_i\sigma_i$$

conservation equations:
one set for each type (matter, radiation, DE, …)

$$\delta_i' = 3(1+w_i)\phi' - \frac{V_i}{Ha^2} - \frac{3}{a}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right)$$

$$V_i' = -(1-3w_i)\frac{V_i}{a} + \frac{k^2}{Ha}\left(\frac{\delta p_i}{\rho_i} + (1+w_i)(\psi - \sigma_i)\right)$$

w, δp, σ: determines physical nature, e.g. cold dark matter: w=δp=σ=0

$$\delta_m' = 3\phi' - \frac{V_m}{Ha^2} \qquad V_m' = -\frac{V_m}{a} + \frac{k^2}{Ha}\psi$$

perturbation evolution

We can (approximately) eliminate V and obtain a second order eqn for δ,

$$\ddot{\delta}_i = -\alpha_i H \dot{\delta}_i + \left(\mu_i H^2 - \frac{c_{s,i}^2 k^2}{a^2} \right) \delta_i$$

α_i, μ_i depend on w_i, c_s^2 is sound speed (<-> δp), 1/3 for r

- α-term: **expansion damping**, may suppress growth
- last term: gravitational collapse vs pressure suppor

  -> will prevent growth if $c_s\, k > Ha$ -> **sound horizo**

  -> with $H^2 = 8\pi G\rho/3$ we have the **Jeans length** λ_J = c

- straightforward to analyze behaviour of matter, rac of scale (horizon, Jeans-length) and of background matter dominated).

???

what happens if $c_s^2<0$?

A) fluctuations disappear

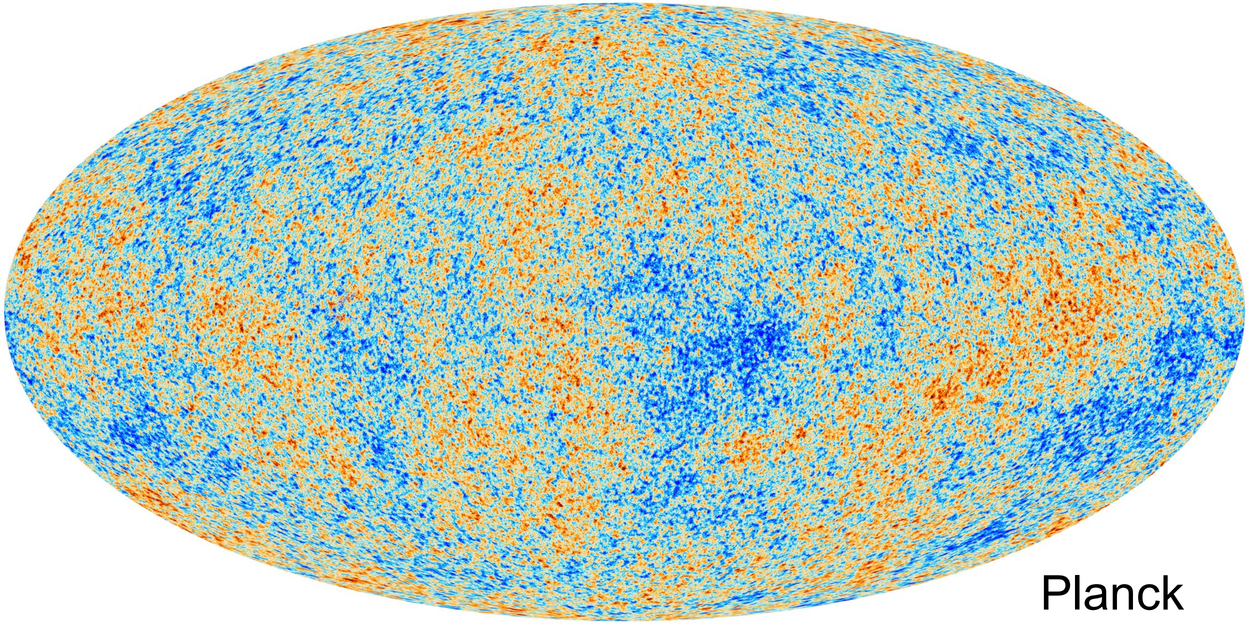
B) fluctuations grow rapidly

C) I don't know

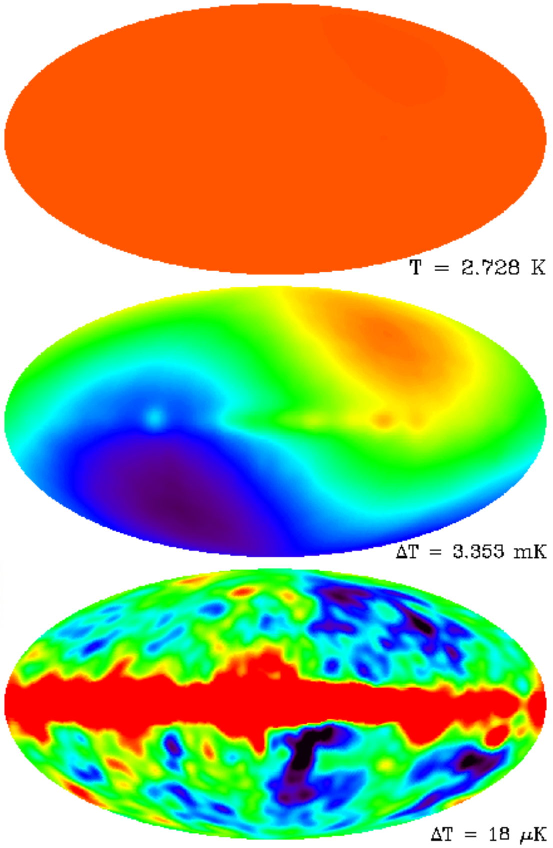
D) I don't even know what the question is about

anisotropies in the CMB

You have often seen this picture
- what does it show?
- why?
- what does it tell us about the universe?



Planck



COBE

origin of the CMB

T > 3000 K :

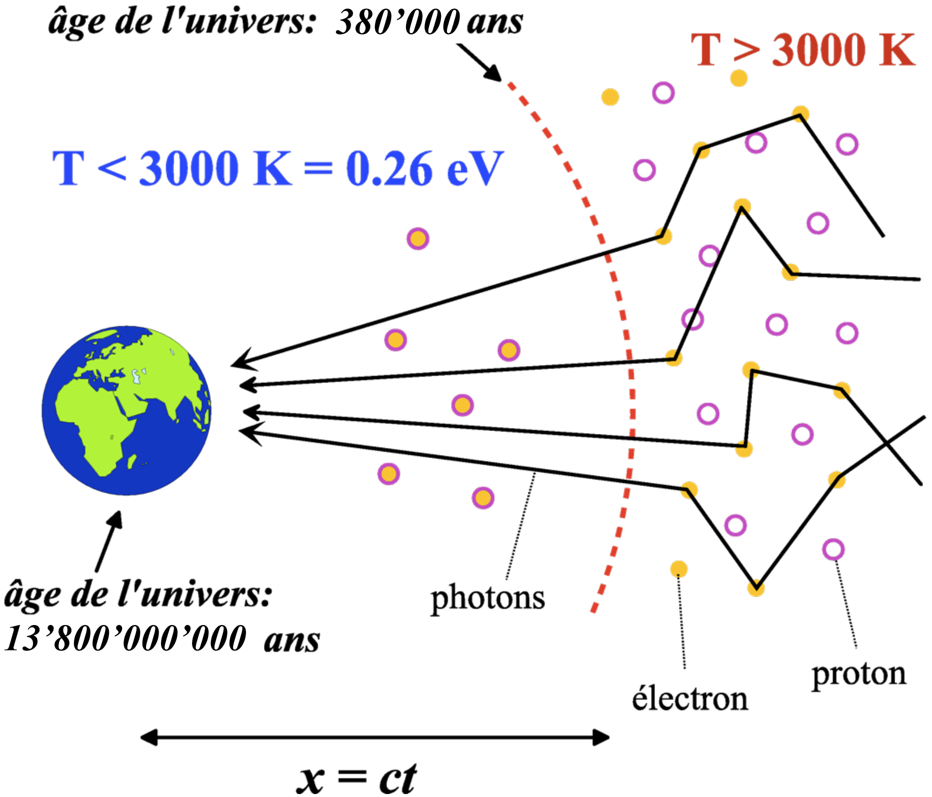
Electrons and protons are free.
Light interacts strongly with the electron (baryon-photon plasma), strong scattering as in fog.

T < 3000 K :

Electrons and protons (re-)combine to neutral atoms.
The universe becomes transparent for light, which free-streams to us.

We observe:

- 'photo' of last scattering surface
- stuff that happens on the way



statistical description

Temperature T(n) on the sky: Gaussian random field

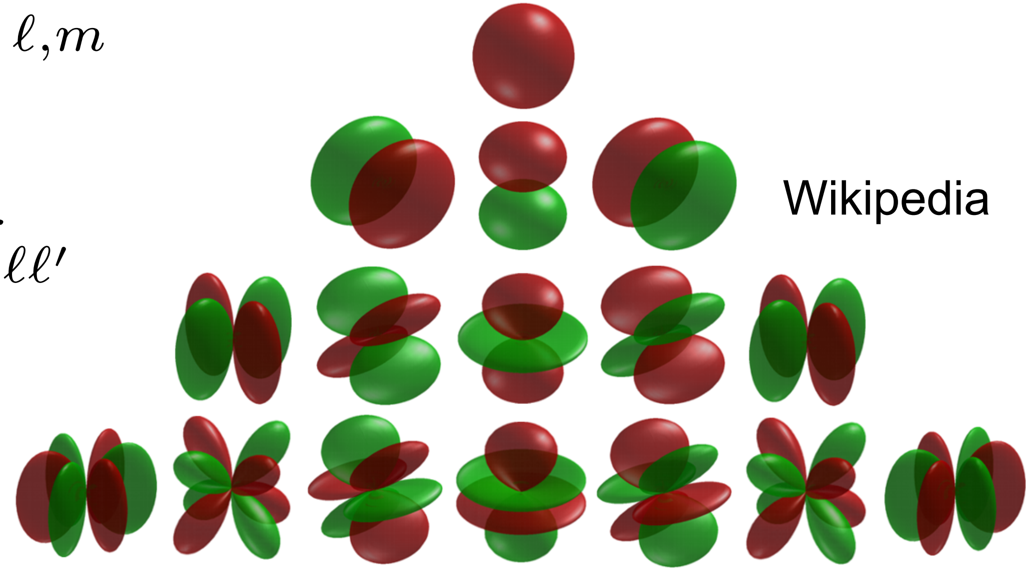
Fourier-analysis on sky sphere: instead of e^{ikt} the basis functions are spherical harmonics $Y_{lm}(n)$

$$\delta T(n) = T(n) - T_0 = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(n)$$

statistical isotropy:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \underbrace{C_\ell}_{\text{power-spectrum} \sim \delta T^2} \delta_{mm'} \delta_{\ell\ell'}$$

power-spectrum
~ δT^2

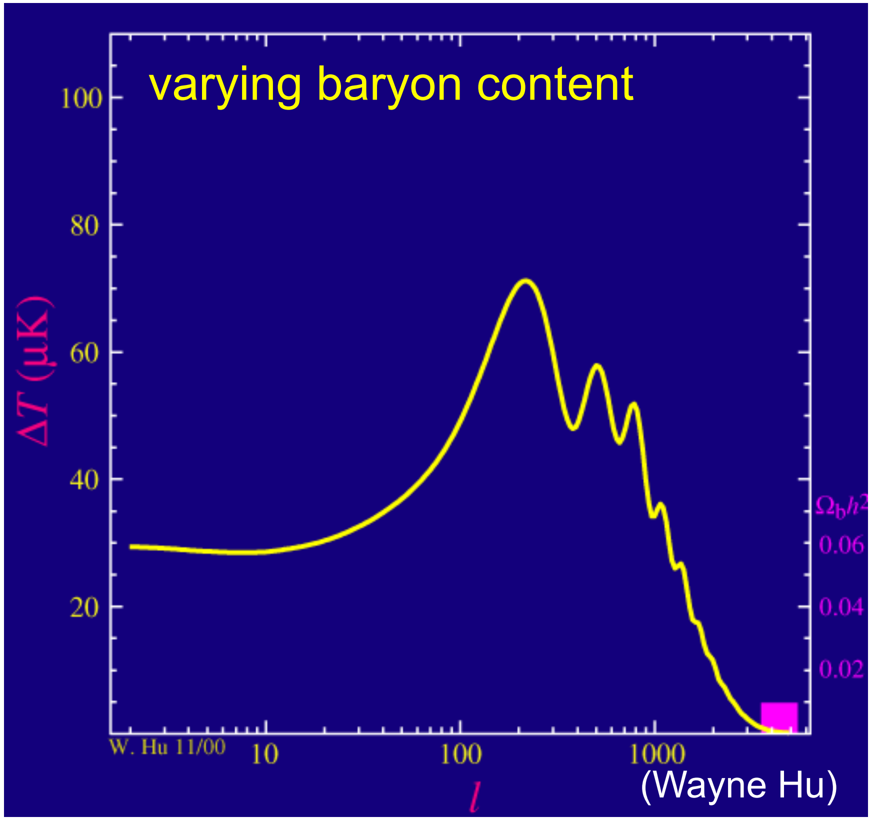


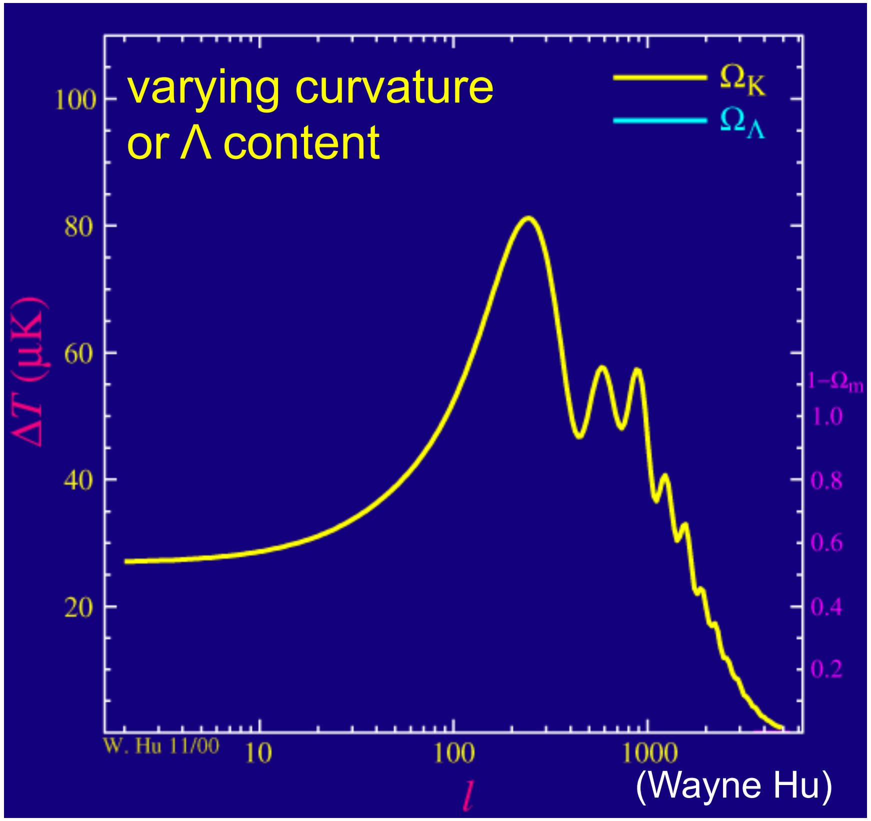
Wikipedia

measuring cosmological parameters

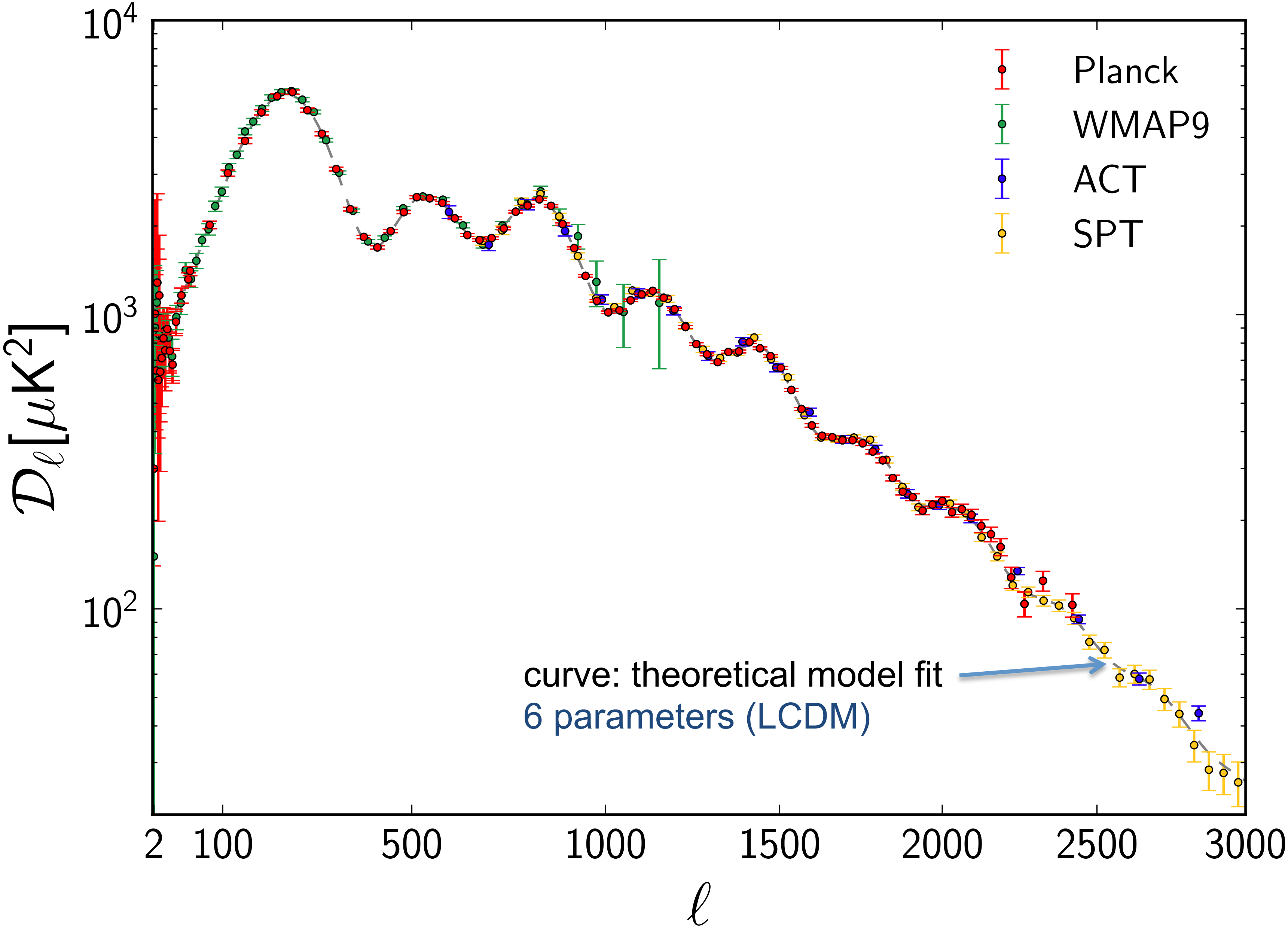
The CMB fluctuations depend on the values of the parameters
→ we just vary all of them to find the best values
(there are public codes for this, e.g. CAMB and CLASS)

CMB physics is mostly linear -> very clean probe!





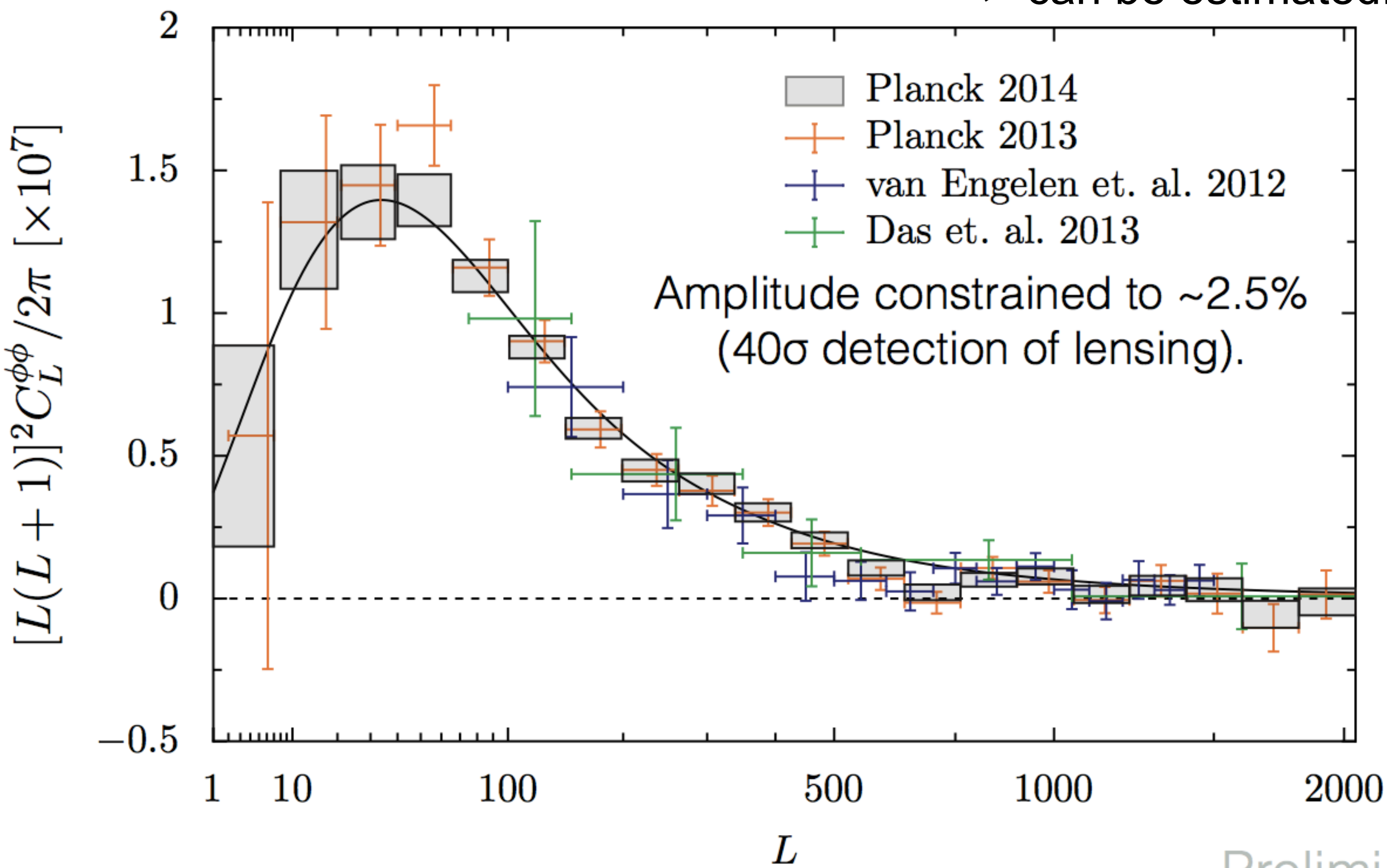
the CMB power spectrum



gravitational lensing of CMB

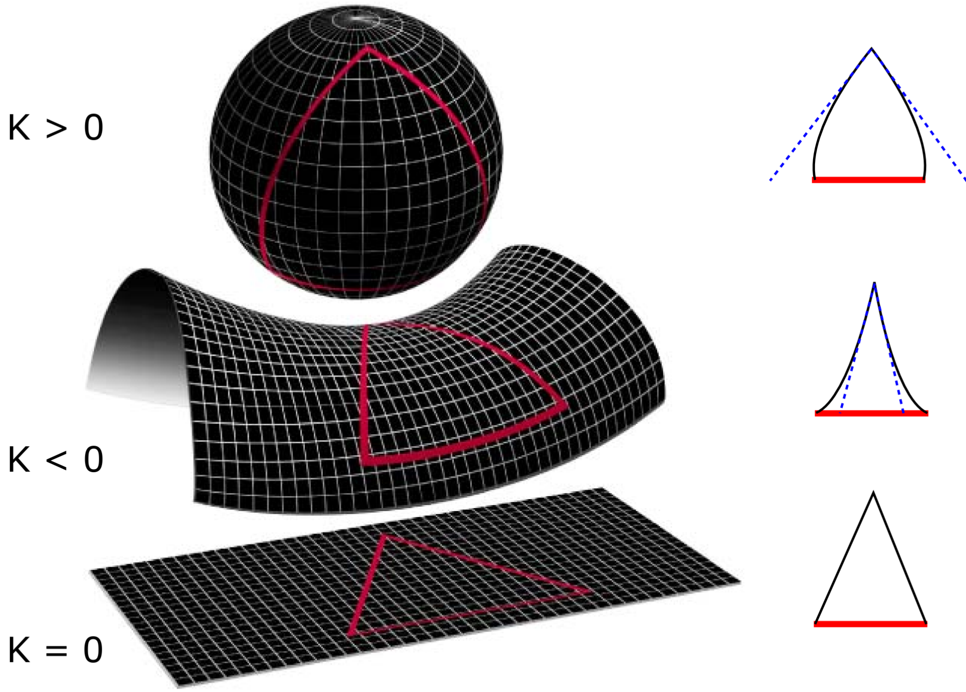
Light is deflected by gravitational perturbations along photon path.

Also true for CMB
-> shifts power around in C_l
-> introduces non-Gaussianity
-> changes polarisation
$\Rightarrow$ can be estimated!



Prelimi

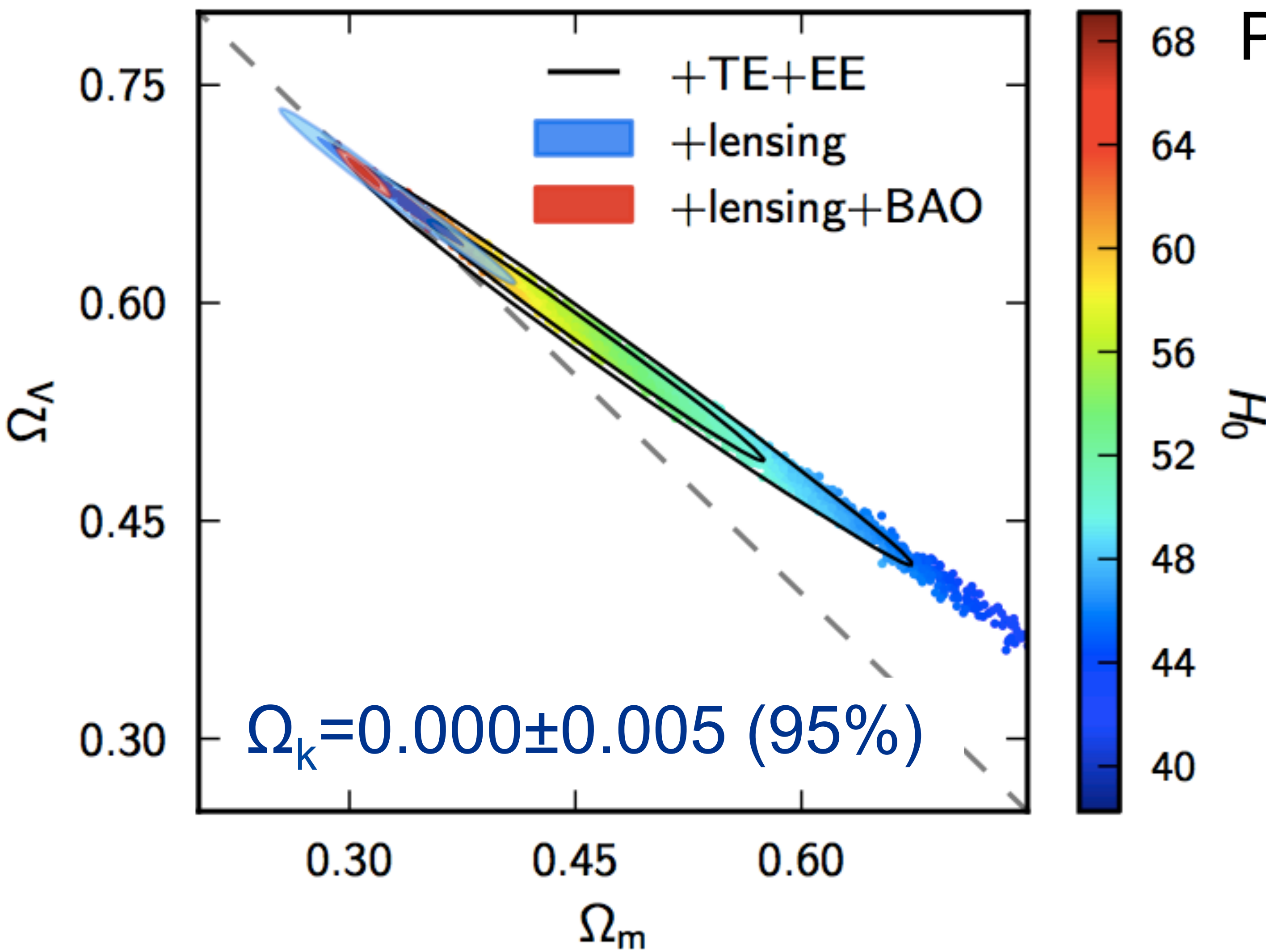
CMB and curvature



The Planck satellite provides ~ 0.03% measurement of the angular scale of the first peak!

-> measurement of the geometry of the universe

how flat is the world?

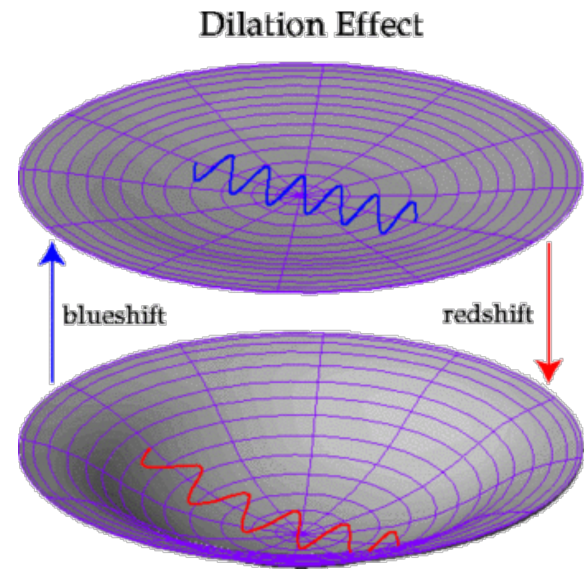


Planck 2015

(integrated) Sachs-Wolfe eff.

Impact of gravitational potential on CMB:

$$\frac{\delta T}{T} \sim (\Phi - \Psi)|_{\mathrm{dec}} + \int_{t_{\mathrm{dec}}}^{t_0} \left(\dot{\Phi} - \dot{\Psi}\right) dt$$



First term: SW -> ~ constant contribution

Second term: ISW -> depends on evolution of the gravitational potential along photon path!

Poisson eq. in matter dom. $\nabla^2\Phi = 4\pi G a^2 \rho_m \delta_m$, ρ_m~a^{-3} , δ_m~a

No ISW effect in a pure matter dominated universe.
But when dark energy begins accelerating the expansion: Φ, Ψ decay
-> ISW provides direct test of accelerated expansion
-> cosmic variance: large uncertainties … about 3σ when correlating with large scale structure

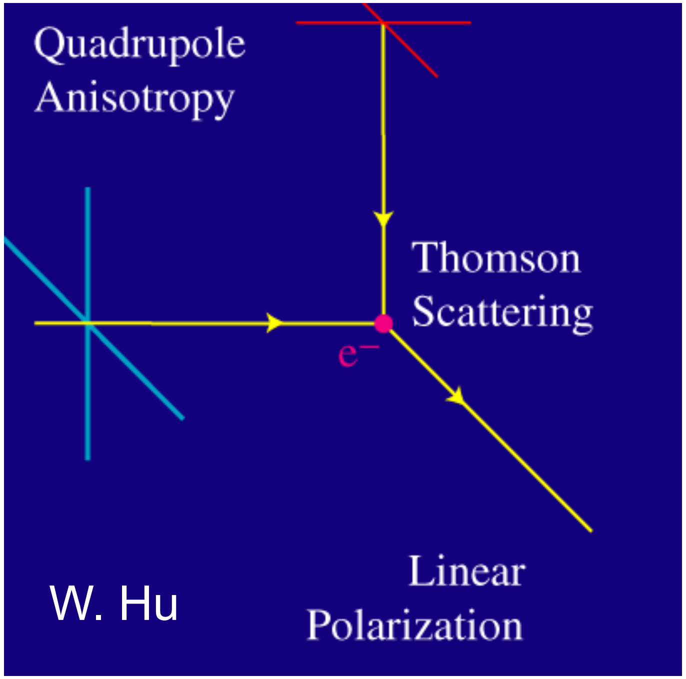
polarization

Scattering of light depends on polarisation angle -> last scattering polarizes light depending on local quadrupole.

-> also reionization probe (scattering again)

Scalar (density) perturbations do not lead to vorticity in polarization pattern ("B-modes")

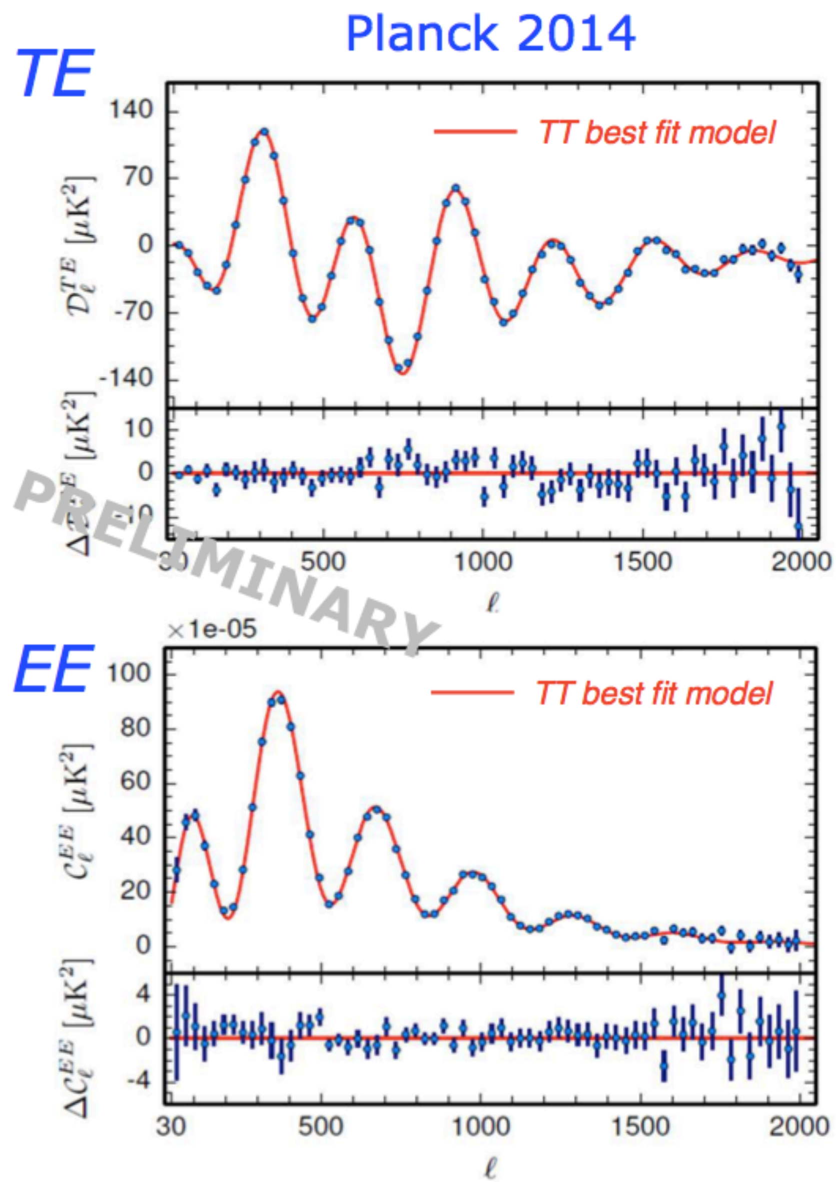
BUT gravitational waves (tensor perturbations) do! (as does lensing)



"B-mode" polarization is a probe of exotic (exciting) physics!

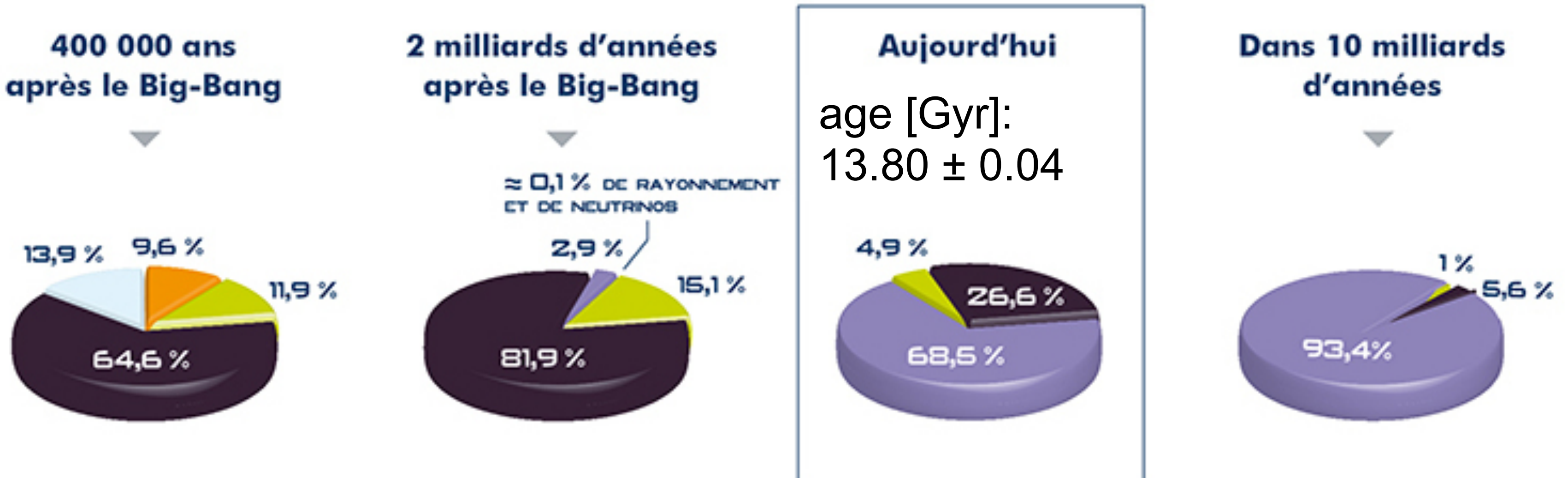
2014 polar power spectrum

- polarisation decomposed into
  - E: gradient type
  - B: vector / rotation type
- for density / scalar perturbations alone, TT predicts TE and EE (and no B-type polarisation)
- CMB lensing and other constituents (e.g. grav. waves) create B-type polarisation
- so do 'foregrounds'
- detection of primordial GW with B-modes would be very important



"precision cosmology"

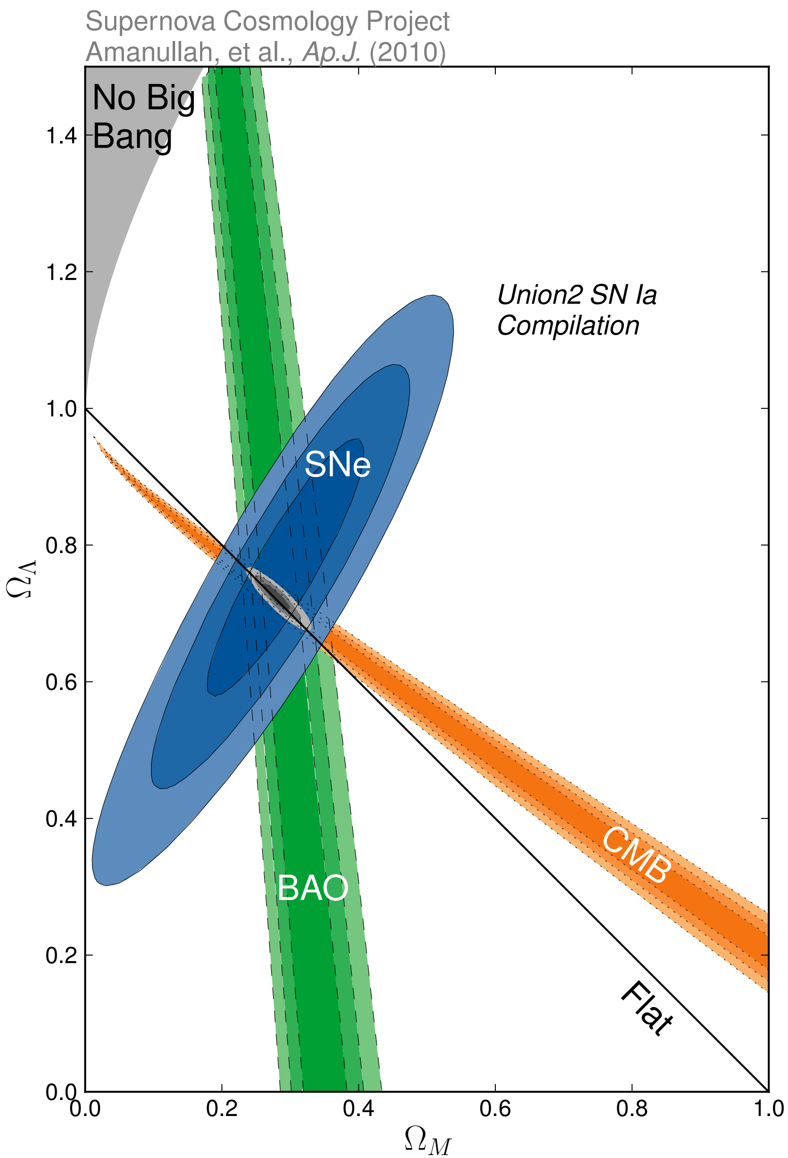
Parameter	[1] *Planck* TT+lowP	[4] *Planck* TT,TE,EE+lowP	
$\Omega_b h^2$	0.02222 ± 0.00023	0.02225 ± 0.00016	$\Omega_b \approx 5\%$
$\Omega_c h^2$	0.1197 ± 0.0022	0.1198 ± 0.0015	
$100\theta_{MC}$	1.04085 ± 0.00047	1.04077 ± 0.00032	0.03% !
τ	0.078 ± 0.019	0.079 ± 0.017	
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.094 ± 0.034	
n_s	0.9655 ± 0.0062	0.9645 ± 0.0049	$n_s \neq 1$
H_0	67.31 ± 0.96	67.27 ± 0.66	
Ω_m	0.315 ± 0.013	0.3156 ± 0.0091	



status report

- we have a full ‘model chain’ that explains cosmological observations
- **the FLRW + LCDM + inflation model is consistent with current data, no significant deviations are observed**
- (some issues with isotropy of the CMB, the structure of galaxies and possibly the growth of perturbations notwithstanding)
- **main problems are theoretical:**
  - we don’t understand 95% of the contents: DE and DM
  - especially the cosmological constant is highly problematic
  - (the model also does not explain how inflation started)
  - (and we can’t explain the baryon asymmetry)

Dark Energy



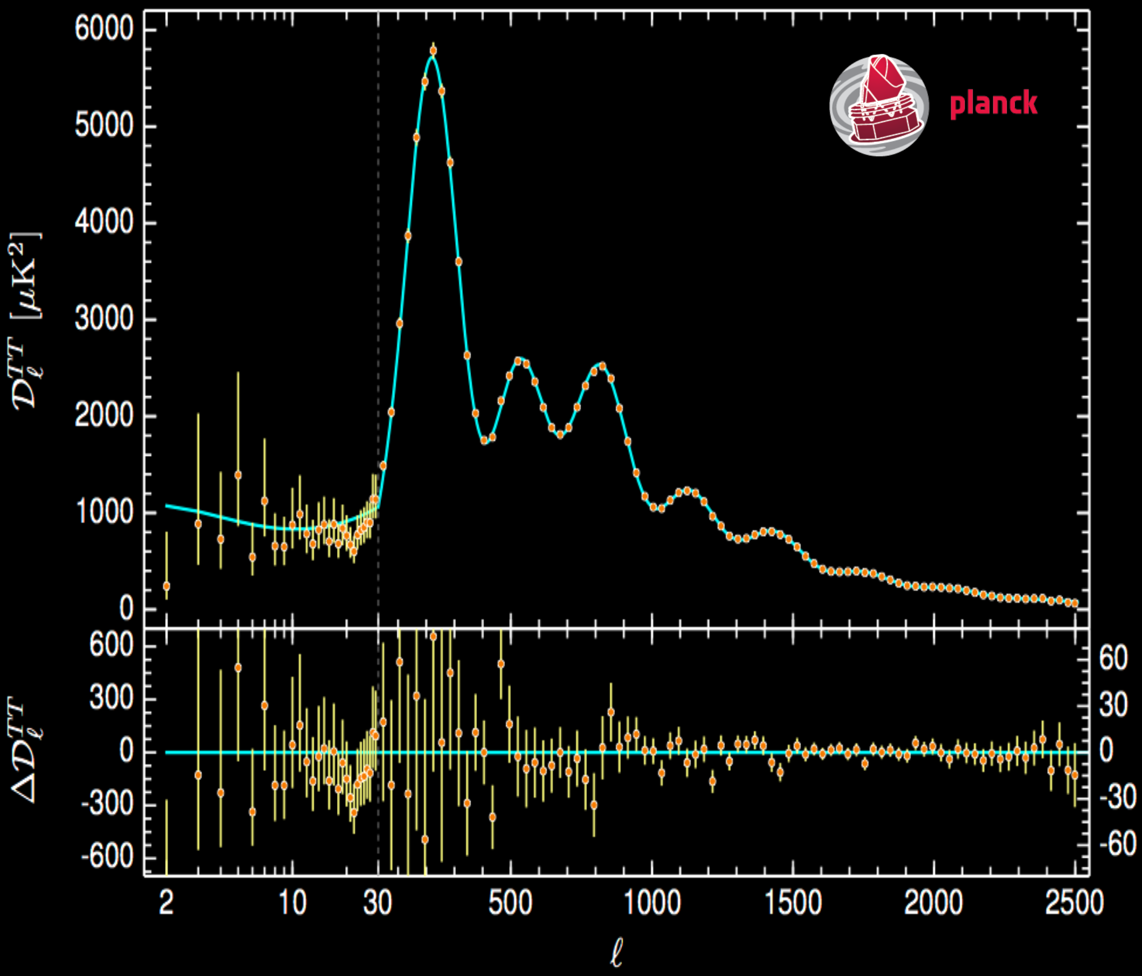
Physics Nobel prize 2011:
"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

accelerating expansion: $w < -1/3$

- we know that for Λ: $w = -1$
- data is consistent with Λ

why look elsewhere?

Planck vs ΛCDM



cyan curve:
best fit 6-parameter flat ΛCDM model

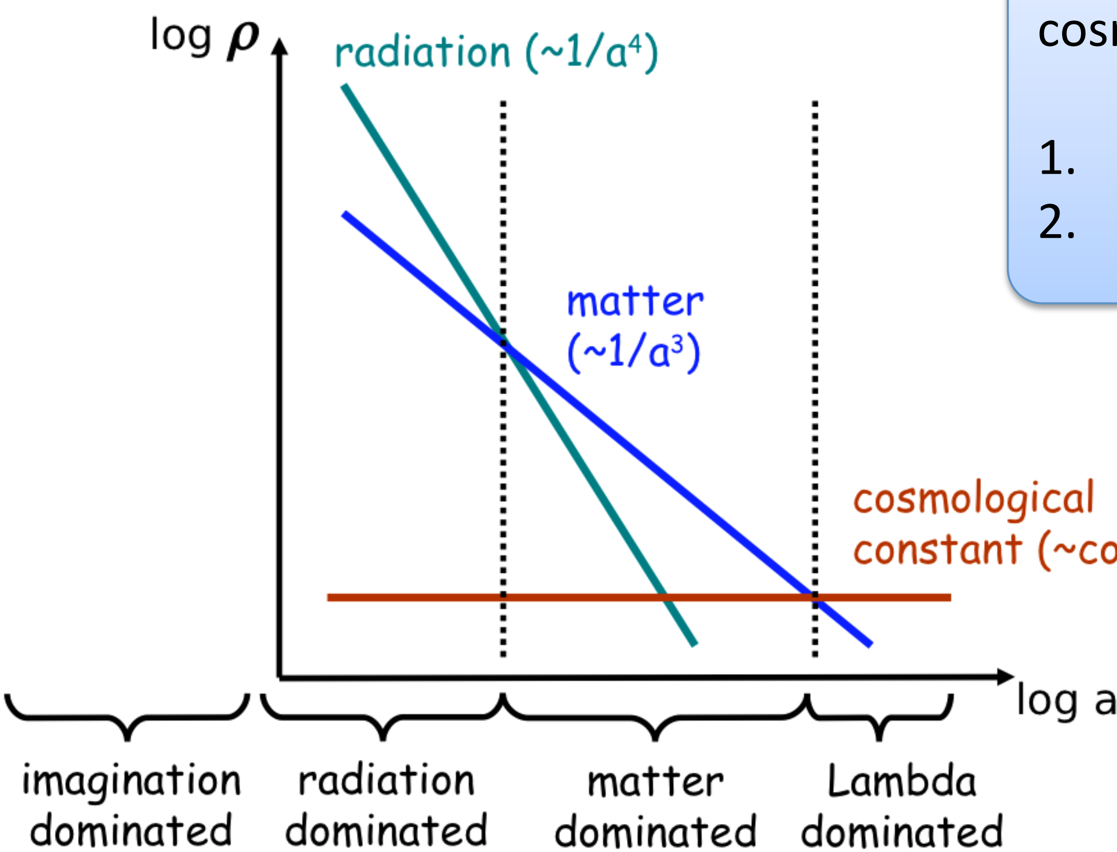
fits millions of CMB pixels (or thousands of C_l)

Planck 2015 TT combined
ell range 30 – 2508
$X^2 = 2546.67$
$N_{dof} = 2479$
PTE 16.8%

reasonable fit except maybe at lowest ell's

What's the problem with Λ?

Evolution of the Universe:

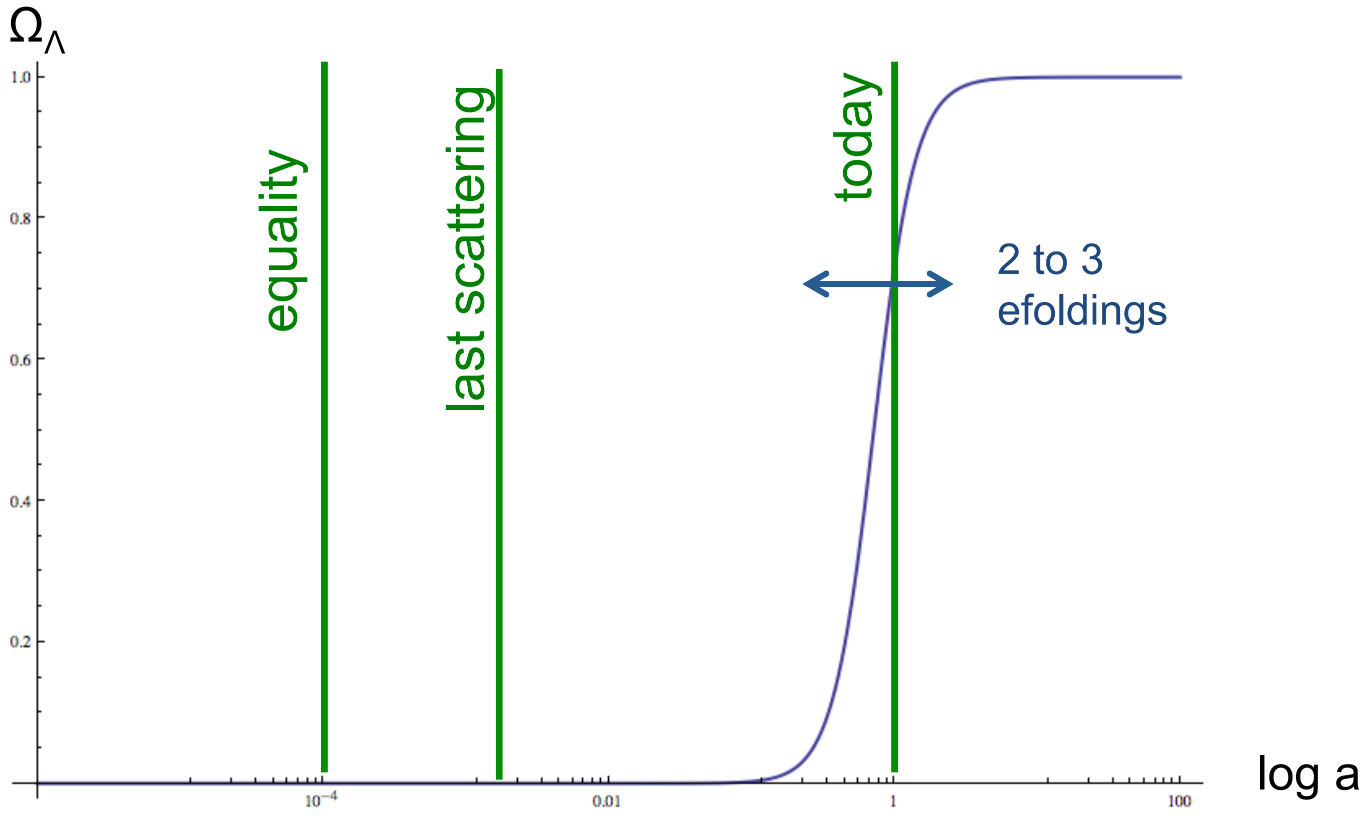


Classical problems of the cosmological constant:

1. Value: why so small? Natural?
2. Coincidence: Why now?

the coincidence problem

- why are we just now observing $\Omega_\Lambda \approx \Omega_m$?
- past: $\Omega_m \approx 1$, future: $\Omega_\Lambda \approx 1$



the naturalness problem

energy scale of observed Λ is ~ 2x10^{-3} eV
zero point fluctuations of a heavier particle of mass m:

$$\int_0^{\overset{\text{cutoff scale}}{\Lambda}} \frac{1}{2}\frac{d^3p}{(2\pi)^3}\sqrt{p^2+m^2} \simeq \frac{1}{16\pi^2}\left[\underbrace{\Lambda^4+m^2\Lambda^2}_{\text{can in principle be absorbed into renormalization of observables}} - \underbrace{\frac{1}{4}m^4\log\left(\frac{\Lambda^2}{m^2}\right)}_{\text{“running” term: this term is measureable for masses and couplings! Why not for cosmological constant?!}}\right]$$

already the electron should contribute at m_e >> eV
(and the muon, and all other known particles!)

Possible explanations

1. It is a cosmological constant, and there is no problem ('anthropic principle', 'string landscape')

2. The (supernova) data is wrong

3. We are making a mistake with GR (aka 'backreaction') or the Copernican principle is violated ('LTB')

4. It is something evolving, e.g. a scalar field ('dark energy')

5. GR is wrong and needs to be modified ('modified gravity')

w during inflation

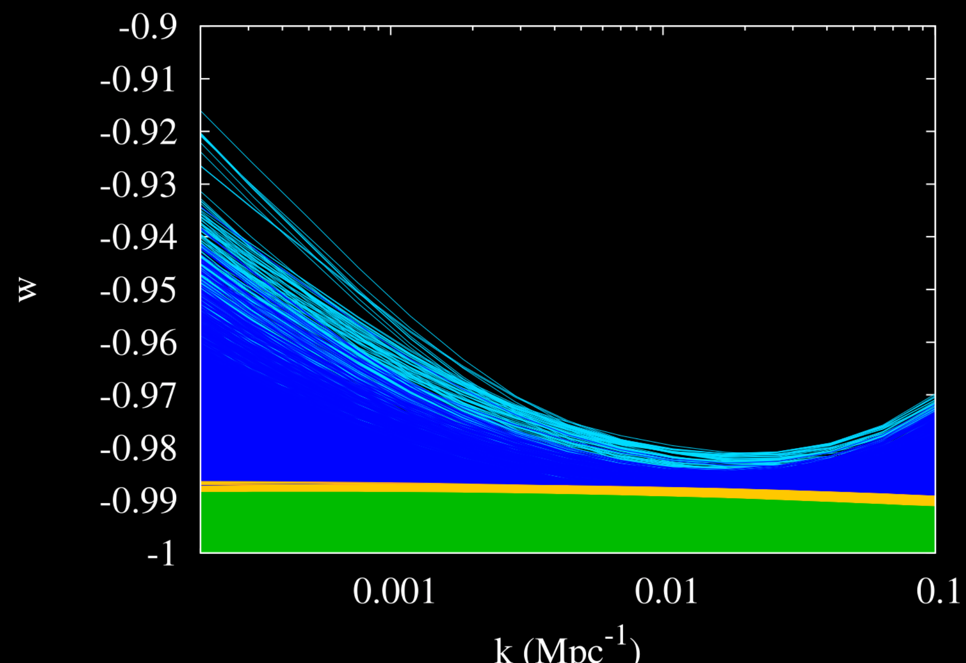
(Ilic, MK, Liddle & Frieman, 2010)

- Scalar field inflaton: $1 + w = -\frac{2}{3}\frac{\dot{H}}{H^2} = \frac{2}{3}\epsilon_H$ and r = T/S ~ 24 (1+w)
- Link to dw/da: $\frac{d\ln(1+w)}{dN} = 2(\eta_H - \epsilon_H)$ $2\eta_H = (n_s - 1) + 4\epsilon_H$

$n_s \neq 1$ => $\varepsilon \neq 0$ or $\eta \neq 0$
=> w ≠ -1 and/or w not constant
=> not a cosmological constant!

WMAP 5yr constraints on w:
- (1+w) < 0.02
- No deviation from w=-1 visible

(but of course not clear if applicable to dark energy)



→ **inflation was not an (even effective) cosmological constant!**
→ inflation is one measurement ahead of dark energy research!

what is the “consensus” 2015?

beyond ΛCDM Oslo 2015

	RD	PL	JM	BR	GS	LV	AH	Beyond LCDM
Dimensions	3+1	3+1	2 in UV	4	4	e^(4-x) x>=4	3+1	3+1
FRW	y	y	n	y	n	y	y	n
Inflation?	y or n	y	n	y	maybe	y	y	y
Dark Matter	CDM	CDM+	none	CDM+	Strange	CDM-like	IDM	Split
Gravity Theory	MG	GRish	Not GR	GR	nearly GR	GR++	GR++	Split
Acceleration: Λ/DE/MG/BR	MG	DE	MG	DE	Λ	Degenerate w/ Λ	Λ	MG
Anomalies =New Physics	n	y	y	n	y	not yet	n	Split

action-based approach

GR +
scalar field:

$$S = S_g + S_\phi = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right)$$

gravity e.o.m.
(Einstein eq.):

$$\boxed{\frac{\delta S[g_{\mu\nu}, \phi]}{\delta g^{\mu\nu}} = 0} \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

scalar field
e.o.m. :

$$\boxed{\frac{\delta S[g_{\mu\nu}, \phi]}{\delta \phi} = 0} \qquad \ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$

Actions specify the model fully

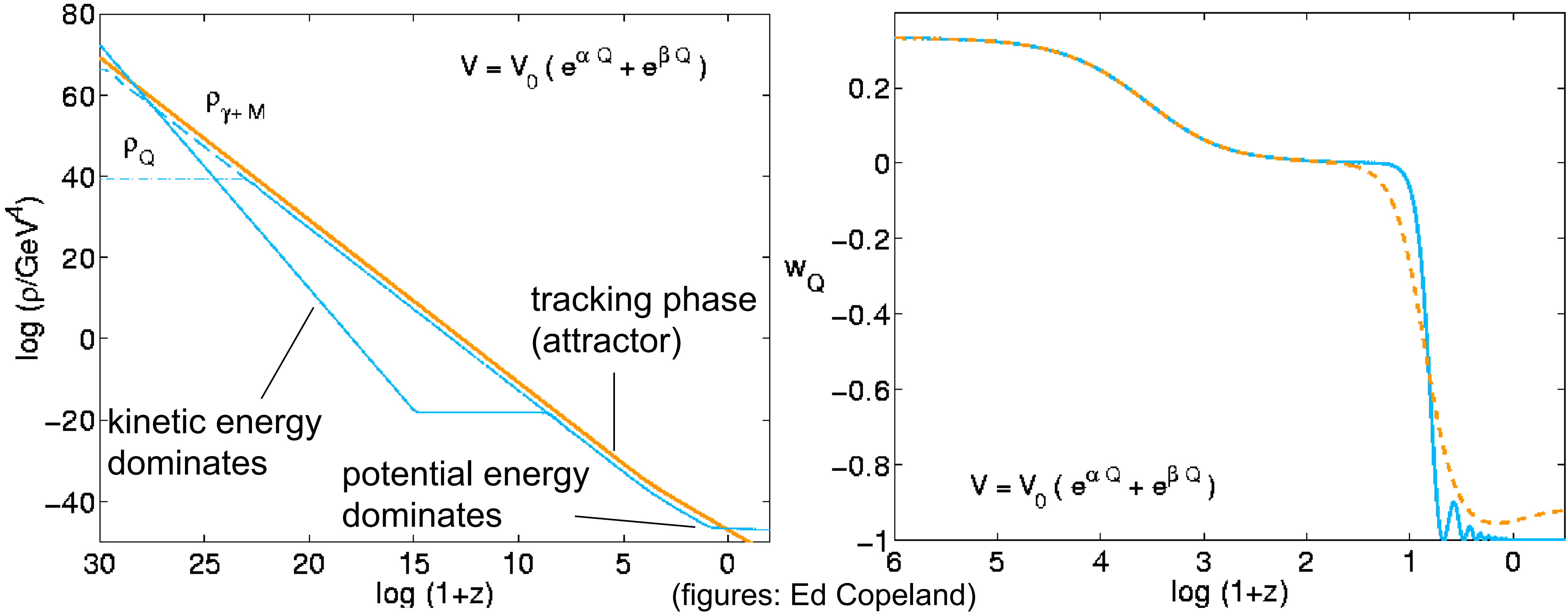
→ but not all properties may be immediately obvious

→ examples: tracking, behaviour in non-linear regime, stability and ghost issues

→ and, of course, we need to specify the action

evolving dark energy

- Inflation: accelerated expansion with help of scalar field
- Dark Energy: accelerated expansion with help of scalar field
- If w=p/ρ can change, then initial dark energy density can be much higher -> solves one problem of Λ
- extra bonus: tracking behaviour



(figures: Ed Copeland)

$$\ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

w = p/ρ

Can write scalar field + 'matter' fluid as dynamical system

-> example for $V(\phi) \propto \exp(-\kappa\lambda\phi)$ $\quad (\kappa^2 = 8\pi G)$

use new variables & write Friedmann and field equations as

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H} \quad N = \ln a \qquad x^2 + y^2 + \frac{\kappa^2\rho_m}{3H^2} = 1$$

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\left[(1-w_m)x^2 + (1+w_m)(1-y^2)\right]$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[(1-w_m)x^2 + (1+w_m)(1-y^2)\right]$$

fixed points (for details see e.g. hep-th/0603057)

1.{x=0,y=0} -> Ω_ϕ=0 (fluid dominated phase)

2.{x=+/-1,y=0} -> Ω_ϕ=1, w_ϕ=1 (kinetic phase)

3.{x=1/sqrt(6),y=[1-λ^2/6]$^{1/2}$} -> Ω_ϕ=1, 1+w_ϕ = $\lambda^2/3$ (dark energy phase)

4.{…} -> $\Omega_\phi = 3(1+w_m)/\lambda^2$, $w_\phi = w_m$ (tracking phase)

Quintessential problems

- no solution to coincidence problem (need to e.g. put a bump into the potential at the right place)
- Still need to get somehow $\Lambda = 0$
- potential needs to be very flat
- need to avoid corrections to potential
- need to avoid couplings to baryons
- no obvious candidates for scalar field (Higgs?)
- but nonetheless quintessence is the 'standard evolving dark energy model'

(***there are many other scalar field models*** – e.g. 'k-essence' and 'growing neutrino' models offer potential solutions to coincidence problem.)

some examples I

(from the Euclid parameter definitions document –
warning: sketchy citations ahead! Please see reviews)

- **quintessence:** minimally coupled canonical scalar field
  - can track background evolution, but cannot avoid fine-tuning
  - could add couplings to gravity and matter

Wetterich 1988
Ratra & Peebles 1988

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + V\right] + S_{\mathrm{matter}}[g]$$

- **K-essence:** generalized kinetic term
  - different clustering (see later), more general tracking

$$\mathcal{L}_\phi = \sqrt{-g}K(\phi, X) \qquad X = \frac{1}{2}(\nabla\phi)^2$$

Armendariz-Picon et al. 2000

some examples II

- **f(R) models:** simplest model with higher derivatives

Weyl 1918?

  - many popular choices for function f

$$\mathcal{L} = \sqrt{-g} f(R)$$

Brans, Dicke 1961

- f(R) is just a **scalar-tensor theory** (universal but non-minimal coupling) after a Legendre transformation Φ~f'
  - Jordan frame and Einstein frame (conformal transf.)
  - S/T theories need to be 'hidden' in the solar system

$$\mathcal{L} = \frac{1}{16\pi}\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\nabla_\mu\phi\nabla^\mu\phi - 2\Lambda(\phi)\right] + \mathcal{L}_m(\Psi, g_{\mu\nu})$$

- **scalar-vector-tensor (eg TeVeS, Aether)**, etc

some examples III

- **Horndeski:** most general theory with 2nd order e.o.m.
  (higher than 2nd order is in general unstable, cf Ostrogradski)

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i$$

Horndeski 1974

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right. \\
&\quad \left. +2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right].
\end{aligned}$$

- popular sub-classes of Horndeski

  Deffayet, Pujolas, Sawicki, Vikman 2010
  - **Kinetic gravity braiding:** most general 'dark energy'
  - **Galileons**

    Nicolis, Rattazzi, Trincherini 2009
- **Effective field theory:** write all operators that are compatible with symmetries (isotropy, homogeneity), single extra scalar – similar to Horndeski, some extra terms?

Creminelli et al 2008
Cheung et al 2008

some examples IV

Hassan, Rosen 2012

de Rham, Gabadadze, Tolley 2010

- **bigravity** and **massive gravity** models
  - very interesting – massive gravity solved 40 year old problem (non-linear completion of Fierz-Pauli)
  - viability and self-consistency still unclear
  - interesting links to other models (e.g. Horneski, Galileons)

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f)$$
$$+ m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$
$$+ \int d^4x\sqrt{-\det g}\mathcal{L}_m\left(g,\Phi\right),$$

- **non-local massive gravity**: viable cosmology w/o direct LCDM limit

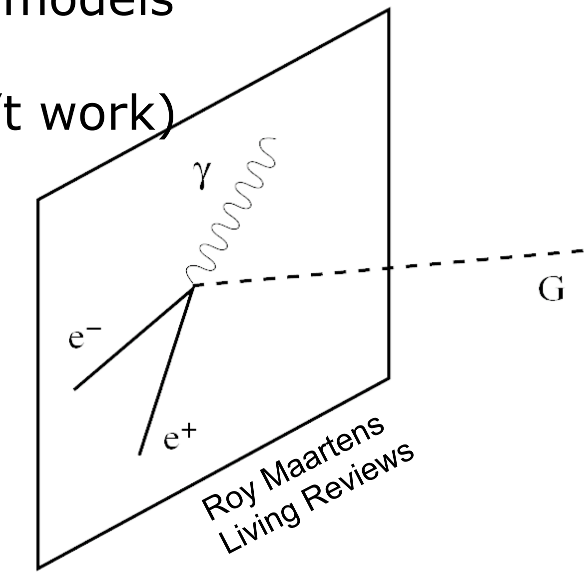
$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R - \frac{1}{6}m^2 R\frac{1}{\Box_g^2}R]$$

Jaccard, Maggiore, Mitsou 2013

some examples MCXIII...?!

e.g. Dvali, Gabadadze, Porrati 2000

- **higher dimensional theories** – typically brane models
  - gravity weakened by leaking into bulk
  - DGP: sum of 4 and 5 dim EH action (doesn't work)
  - 6+ dimensions (may work?)
  - rewrite as 4D effective model
    → EFT / Horndeski



Roy Maartens
Living Reviews

- we could go on a for a while ...

Many more examples (apologies if I did not mention your favourite theory ☹ ; read a review for details! ☺) ... some approaches (Horndeski/EFT) are very general, but are they general enough? Can we do something else to look for deviations from LCDM?

→ phenomenological approach based on evolution of the geometry and/or properties of the effective dark energy fluid

non-cosmological probes

- fifth force (weak, long-range) from couplings of standard model to new fields

-> screening mechanisms (Chameleon, Vainshtein, …)

- new particles with strange couplings and/or mass hierarchies (KK)
- varying "fundamental constants" and other violations of the equivalence principle
- perihelion shifts / solar system constraints (including double pulsar timings, etc)
- modifications to stellar structure models
- short-distance gravity modified (now well below 0.1mm)

Einstein vs Jordan frames

$$g_{\mu\nu} = e^{2f}\tilde{g}_{\mu\nu}$$

f(R) Jordan frame
universally coupled but strange gravity

$$\mathcal{L} = -f(\varphi)R + \mathcal{L}_m[m]$$

$$\mathcal{L}_{,R}G_{\mu\nu} = 8\pi G T_{\mu\nu} + \mathcal{F}[\mathcal{L}, f]$$

$$T^{\mu}_{(m)\nu;\mu} = 0$$

$$T^{\mu}_{(\varphi)\nu;\mu} = 0$$

f(R) Einstein frame
GR but coupled DE

$$\mathcal{L} = -R + \mathcal{L}_m[\varphi, m]$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T^{\mu}_{(m)\nu;\mu} = C T_{(m)}\varphi_{,\nu}$$

$$T^{\mu}_{(\varphi)\nu;\mu} = -C T_{(m)}\varphi_{,\nu}$$

screening

e.g. Khoury arXiv:1011.5909

- universally coupled scalar d.o.f. → 5th force
- needs to be hidden in the solar system, or model ruled out
- interestingly, many have generic mechanisms to do just do that

schematic Lagrangian in Einstein frame:

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \partial^2\phi)\partial_\mu\phi\partial_\nu\phi - V(\phi) + \beta(\phi) T^\mu_\mu$$

- matter EMT can give dependence on local density

1. **chameleon mechanism**: large mass in high-density region, Yukawa force leads to short-range effects only
2. **symmetron/dilaton** mechanism: small coupling in high-density region
3. **k-mouflage/Vainshtein** mechanism: large kinetic function Z (large derivatives) in high-density region to suppress effective coupling to matter

- needs numerical simulations → not easy for future surveys like Euclid
- (small scales also have other issues like baryons)

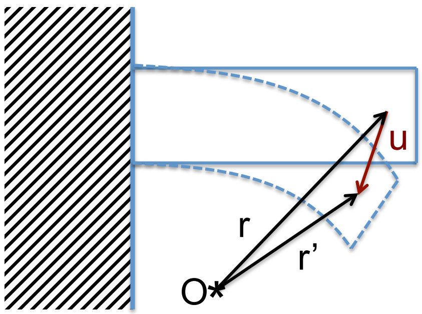
status report

- Data tells us that we need something more than just the standard model of particle physics
- A cosmological constants seems to fit
- But we have to consider also alternatives
  - ‘classical’ problems of cosmological constant
  - inflation looks a bit like dynamical dark energy
  - need to know against what we should compare LCDM
- The problem is not that there are no models … ☺
- Is there a systematic approach?

effective (field) theories

- model observations on scales of interest
- ignore degrees of freedom on much smaller scales
- example: fluid dynamics where we model a fluid in terms of density ρ, pressure p and velocity field **v** without caring about the physical atoms that make up the fluid
- typically needs a separation of scales
- examples of effective QFT's that worked well:
  - Fermi theory of the weak interaction where W and Z are integrated out and we have four-fermion interactions, works for E < 100 GeV
  - Chiral perturbation theory for low-energy dynamics of QCD, where gluons are replaced by pion mediated interactions
- EFT's are often non-renormalizable
- no problem, they are not fundamental theories!

effective theory of elasticity



- either build detailed model at molecular level
- or effective model of deformations

effective d.o.f → deformation tensor:

$$u_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

expand energy in terms of deformation:

$$E = E_0 + \overbrace{\lambda_1^{ij}(\mathbf{x})}^{\text{vanishes because of definition of 'no deformation'}} u_{ij} + \underbrace{\lambda_2^{ijkl}(\mathbf{x})}_{\left.\frac{\partial^2 E}{\partial u_{ij} \partial u_{kl}}\right|_0} u_{ij} u_{kl} + \ldots$$

apply symmetry constraints: isotropy + homogeneity → λ_2 scalar const.

$$E = E_0 + \frac{1}{2}\lambda \left(u_i^i\right)^2 + \mu u_{ij} u^{ij} = E_0 + \frac{1}{2}K \left(u_i^i\right)^2 + \mu \left(u_{ij} - \frac{1}{3}\delta_{ij} u_k^k \right)^2$$

lessons:
- valid only in a certain regime (eg. no big deformations, not crystals)
- can be written in different ways, some better for physical interpretation

effective theory of dark energy

different approaches, but generally:

- define 3+1 split (FLRW or based on uniform scalar field hypersurfaces)
- geometry can then be described by ${}^3R_{\mu\nu}$, extrinsic curvature $K_{\mu\nu}$, g^{00} or N
- now expand action (e.g. Gleyzes et al, 2015)

$$L(N, K^i_j, R^i_j, \dots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j}\delta K^i_j + \frac{\partial L}{\partial R^i_j}\delta R^i_j + L^{(2)} + \dots,$$

$$\begin{aligned} L^{(2)} =& \frac{1}{2}L_{NN}\delta N^2 + \frac{1}{2}\frac{\partial^2 L}{\partial K^i_j \partial K^k_l}\delta K^i_j \delta K^k_l + \frac{1}{2}\frac{\partial^2 L}{\partial R^i_j \partial R^k_l}\delta R^i_j \delta R^k_l + \\ &+ \frac{\partial^2 L}{\partial K^i_j\, \partial R^k_l}\delta K^i_j \delta R^k_l + \frac{\partial^2 L}{\partial N \partial K^i_j}\delta N \delta K^i_j + \frac{\partial^2 L}{\partial N \partial R^i_j}\delta N \delta R^i_j + \dots \end{aligned}$$

the coefficients can be collected in different ways, impose isotropy & homogeneity & conditions to ensure no more than 2nd derivatives in e.o.m.

impose conditions

(Cheung et al, 2008; Gubitosi et al, 2013, Bloomfield et al 2013, … below is Bloomfield used in in EFTcamb, contains also higher derivatives, models w/Lorentz violation)

$$
\begin{aligned}
S \;=\; \int d^4x\sqrt{-g}\Bigg\{ & \frac{m_0^2}{2}\left[1+\Omega(\tau)\right]R+\Lambda(\tau)-a^2c(\tau)\delta g^{00} \\
& +\frac{M_2^4(\tau)}{2}\left(a^2\delta g^{00}\right)^2-\bar{M}_1^3(\tau)2a^2\delta g^{00}\delta K_\mu^\mu \\
& -\frac{\bar{M}_2^2(\tau)}{2}\left(\delta K_\mu^\mu\right)^2-\frac{\bar{M}_3^2(\tau)}{2}\delta K_\nu^\mu\delta K_\mu^\nu+\frac{a^2\hat{M}^2(\tau)}{2}\delta g^{00}\delta R^{(3)} \\
& +m_2^2(\tau)\left(g^{\mu\nu}+n^\mu n^\nu\right)\partial_\mu\left(a^2g^{00}\right)\partial_\nu\left(a^2g^{00}\right)\Bigg\} \\
& +S_{\mathrm{m}}[\chi_i,g_{\mu\nu}]. \qquad (2)
\end{aligned}
$$

a compact notation is (Bellini & Sawicki 2014, Gleyzes et al 2015)

$$
\begin{aligned}
S^{(2)} = \int d^3x\,dt\,a^3\,\frac{M^2}{2}\Bigg[& \delta K_{ij}\delta K^{ij}-\delta K^2+(1+\alpha_T)\left(R\frac{\delta\sqrt{h}}{a^3}+\delta_2 R\right) \\
& +\alpha_K H^2\delta N^2+4\alpha_B H\,\delta K\,\delta N+(1+\alpha_H)R\,\delta N\Bigg]
\end{aligned}
$$

→ 6 free coefficients: $M(t)$ [or $\alpha_M(t)$], $\alpha_T(t)$, $\alpha_K(t)$, $\alpha_B(t)$, $\alpha_H(t)$ and $H(t)$

interpretation of EFT d.o.f.

(mostly Bellini & Sawicki 2014)

- H(t): background evolution
- $\alpha_K(t)$: "kineticity" – kinetic energy, large $\alpha_K \rightarrow$ small c_s^2;
- $\alpha_B(t)$: "braiding" – mixing of kinetic terms and metric, contributes to DE clustering
- $\alpha_M(t)$: "Planck mass run rate", $\alpha_M=1/(2H)\, d(\ln M^2)/dt$, contributes to anisotropic stress
- $\alpha_T(t)$: "tensor speed excess", also contributes to anisotropic stress
- $\alpha_H(t)$: "beyond Horndeski", higher order term in Einstein eq. that cancels in e.o.m.

These are 'properties of the material' (i.e. dark energy) and to be measured from data, there is no a priori hierarchy in EFT's

there are also stability conditions on the α_i like $c_s^2>0$, $c_T^2>0$, positive kinetic terms, cf Bellini&Sawicki 2014

link to scalar fields/Horndeski

(Bloomfield 2013, here following again Gleyzes 2015)

Use 'Stückelberg trick' to restore general covariance and reintroduce scalar field perturbations

$t \rightarrow t+\pi(t,x)$; $\varphi = \varphi_0(t+\pi) = \varphi_0(t) + \delta\varphi$

the functions then transform as

$$f \to f + \dot{f}\pi + \frac{1}{2}\ddot{f}\pi^2 + \mathcal{O}(\pi^3) \quad \Rightarrow \quad \delta\varphi = \dot{\varphi}\pi$$

and the `Stückelberg field' π e.o.m. is

$$\begin{aligned}
&H^2\alpha_K\ddot{\pi} + \left\{\left[H^2(3+\alpha_M) + \dot{H}\right]\alpha_K + (H\alpha_K)^{\cdot}\right\}H\dot{\pi} \\
&+ 6\left\{\left(\dot{H} + \frac{\rho_{\rm m}+p_{\rm m}}{2M^2}\right)\dot{H} + \dot{H}\alpha_B\left[H^2(3+\alpha_M)+\dot{H}\right] + H(\dot{H}\alpha_B)^{\cdot}\right\}\pi \\
&- 2\frac{k^2}{a^2}\left\{\dot{H} + \frac{\rho_{\rm m}+p_{\rm m}}{2M^2} + H^2\left[1+\alpha_B(1+\alpha_M)+\alpha_T-(1+\alpha_H)(1+\alpha_M)\right] + (H(\alpha_B-\alpha_H))^{\cdot}\right\}\pi \\
&+ 6H\alpha_B\ddot{\Psi} + H^2(6\alpha_B-\alpha_K)\dot{\Phi} + 6\left[\dot{H} + \frac{\rho_{\rm m}+p_{\rm m}}{2M^2} + H^2\alpha_B(3+\alpha_M) + (\alpha_B H)^{\cdot}\right]\dot{\Psi} \\
&+ \left[6\left(\dot{H} + \frac{\rho_{\rm m}+p_{\rm m}}{2M^2}\right) + H^2(6\alpha_B-\alpha_K)(3+\alpha_M) + 2(9\alpha_B-\alpha_K)\dot{H} + H(6\dot{\alpha}_B-\dot{\alpha}_K)\right]H\Phi \\
&+ 2\frac{k^2}{a^2}\left\{\alpha_H\dot{\Psi} + \left[H(\alpha_M+\alpha_H(1+\alpha_M)-\alpha_T)-\dot{\alpha}_H\right]\Psi + (\alpha_H-\alpha_B)H\Phi\right\} = 0\,.
\end{aligned}$$

for α_H=0 this is equivalent to Horndeski (Bloomfield 2013), but the EFT approach is explicitly *NOT* supposed to be a 'fundamental theory'!

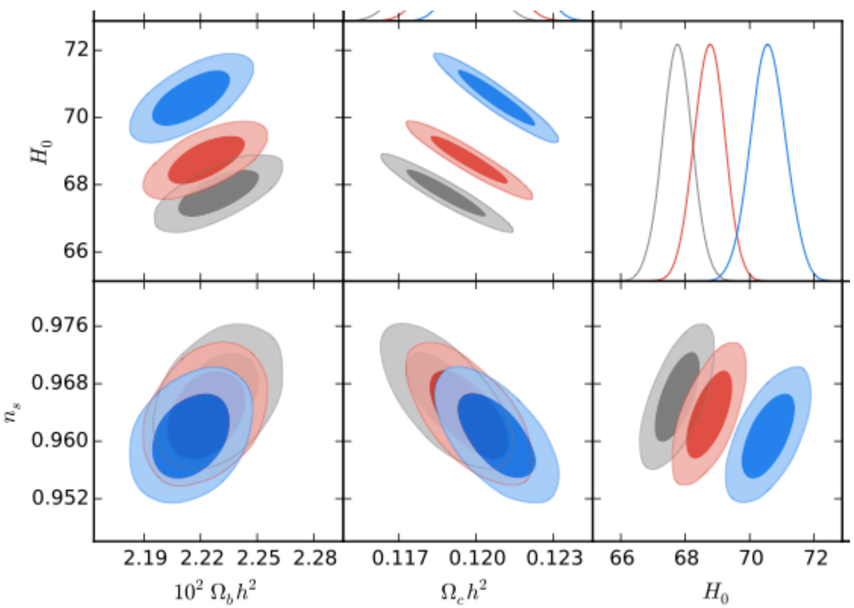
brief aside on non-local models

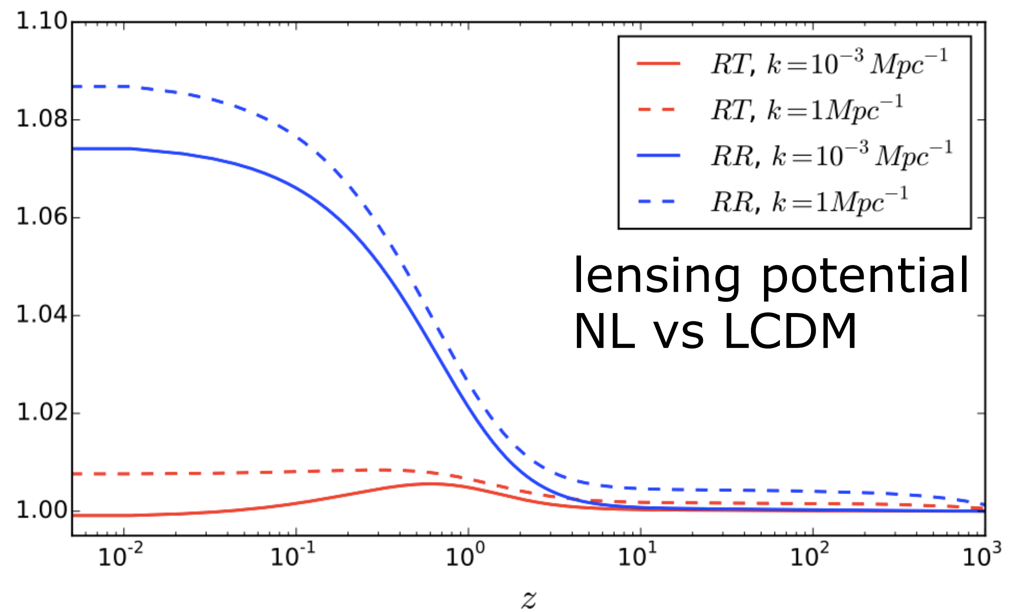
some model classes are not reflected in EFT, e.g. non-local models like (Maggiore et al), models with torsion, non-metric theories, Palatini, …

$$S = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{6}m^2 R\frac{1}{\Box^2}R\right]$$

important on large scales

Non-local models are themselves effective models, unlikely to be fundamental. Nice aspect: different from LCDM but fitting the data as well – Euclid will be able to tell them apart, good benchmark





action-based approach

GR + scalar field:

$$S = S_g + S_\phi = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \boxed{V(\phi)}\right)$$

gravity e.o.m. (Einstein eq.):

$$\boxed{\frac{\delta S[g_{\mu\nu},\phi]}{\delta g^{\mu\nu}} = 0}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot\phi^2 - V(\phi)$$

$$\boxed{w = p/\rho}$$

scalar field e.o.m. :

$$\boxed{\frac{\delta S[g_{\mu\nu},\phi]}{\delta\phi} = 0}$$

$$\ddot\phi + 3H\dot\phi + dV(\phi)/d\phi = 0$$

- The equation of motion of Φ corresponds to a fluid with certain parameters (sound speed = speed of light, no anisotropic stress)
- The free function V(Φ) corresponds to a choice of w(z) or H(z)
- Can we bypass the field-based model and look at w or H directly? This eliminates possible degeneracies with observations too!

phenomenology of the dark side

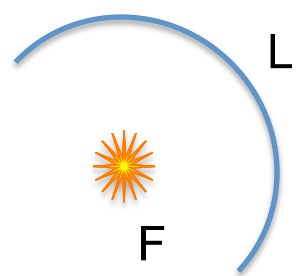
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

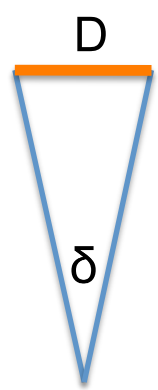
(determined by the metric) geometry

stuff (what is it?)

your favourite theory

distances $d \sim \int_0^z \frac{dz}{H(z)}$





$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

"effective" scalar field fluids

How about perturbations? **It works too!**

$$\delta_i' = 3(1+w_i)\phi' - \frac{V_i}{Ha^2} - \frac{3}{a}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right)$$

$$V_i' = -(1-3w_i)\frac{V_i}{a} + \frac{k^2}{Ha}\left(\frac{\delta p_i}{\rho_i} + (1+w_i)(\psi - \sigma_i)\right)$$

Newtonian gauge fluid perturbation equations

$$-\delta T^0_0 = \delta\rho = \frac{1}{a^2}\dot{\phi}\dot{\delta\phi} - \frac{1}{a^2}\dot{\phi}^2\Psi + \frac{dV}{d\phi}\delta\phi$$

$$\delta T^i_i = \delta p = \frac{1}{a^2}\dot{\phi}\dot{\delta\phi} - \frac{1}{a^2}\dot{\phi}^2\Psi - \frac{dV}{d\phi}\delta\phi$$

$$-ik\delta T^i_0 = ik\delta T^0_i = \frac{k^2}{a^2}\dot{\phi}\delta\phi = \bar{\rho}V$$

"dictionary" from

$$\frac{\delta S[g_{\mu\nu},\phi]}{\delta g^{\mu\nu}} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\ddot{\delta\phi} + 2aH\dot{\delta\phi} + a^2\left(\frac{d^2V}{d\phi^2} + \frac{k^2}{a^2}\right)\delta\phi = 4\dot{\phi}\dot{\Psi} - 2a^2\Psi\frac{dV}{d\phi}$$

perturbation e.o.m. from $\dfrac{\delta S[g_{\mu\nu},\phi]}{\delta\phi} = 0$

“effective” scalar field fluids

What is the equivalent model?

- Introduce rest-frame sound speed

  $\delta p = c_s^2 \, \delta\rho$

- gauge transformation to Newtonian gauge

$$\delta p = \hat{c}_s^2 \delta\rho + \frac{3aH}{k^2}\left(\hat{c}_s^2 - c_a^2\right)\bar{\rho}V$$

- magic correspondence: evolution of linear scalar field perturbations correspond to fluid with

  $c_s^2=1$, $\sigma=0$

- e.g. K-essence is generalization to arbitrary $c_s^2 = K_{,X}/(K_{,X}+2XK_{,XX})$ (and KGB to more complicated δp)
- physics determines how much freedom is in functions

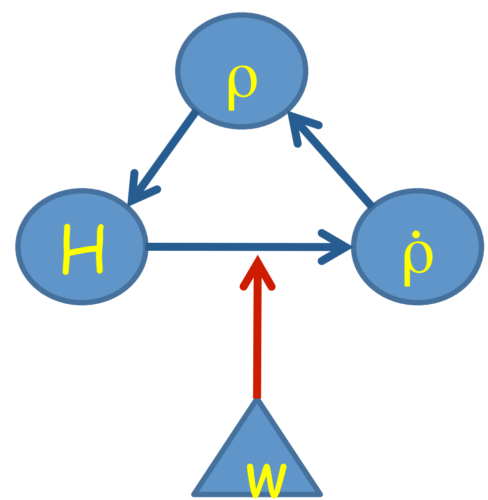
the background case

$$ds^2 = -dt^2 + a(t)^2 dx^2$$ metric "template"

Einstein eq'n $$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_1 + \rho_2 + \ldots + \rho_n)$$

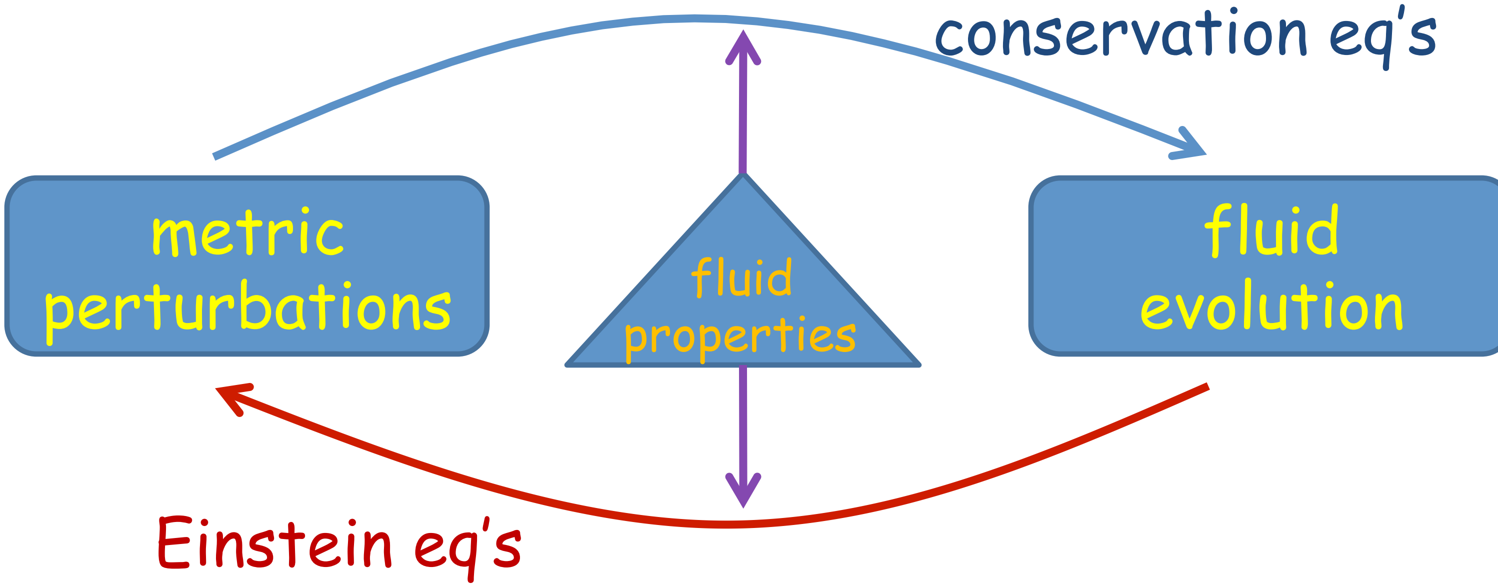
conservation $$\dot{\rho}_i = -3H(\rho_i + p_i) = -3H(1 + w_i)\rho_i \qquad i = 1, \ldots, n$$

- w_i describe the fluids
- normally all but one known
- H|a describe observables (distances, ages, etc)



perturbations

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2$$ metric (gauge fixed, scalar dof)



$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \left(\delta_i + 3Ha\frac{V_i}{k^2}\right), \quad k^2(\phi-\psi) = 12\pi G a^2 \sum_i (1+w_i)\rho_i\sigma_i$$

$$\delta_i' = 3(1+w_i)\phi' - \frac{V_i}{Ha^2} - \frac{3}{a}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right)$$

$$V_i' = -(1-3w_i)\frac{V_i}{a} + \frac{k^2}{Ha}\left(\frac{\delta p_i}{\rho_i} + (1+w_i)(\psi-\sigma_i)\right)$$

general dark phenomenology

modified "Einstein" eq:
(projection to 3+1D)

$$X_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$\boxed{G_{\mu\nu} = -8\pi G T_{\mu\nu} - Y_{\mu\nu} \qquad Y_{\mu\nu} \equiv X_{\mu\nu} - G_{\mu\nu}}$$

$Y_{\mu\nu}$ can be seen as an effective DE energy-momentum tensor.

Is it conserved?

Yes, since $T_{\mu\nu}$ is conserved, and since $G_{\mu\nu}$ obeys the Bianchi identities!

Cosmology can measure effective DE EMT

the geometric EMT

(G. Ballesteros, L. Hollenstein, R. Jain & MK)

$$1 + w_G = -\frac{2}{3}\frac{\dot{H}}{H^2}$$

$$\delta\rho_G = -2M_P^2\left[3H\left(\dot{\phi} + H\psi\right) - a^{-2}\nabla^2\phi\right]$$

$$\delta p_G = 2M_P^2\left[\ddot{\phi} + H\left(3\dot{\phi} + \dot{\psi}\right) - 3w_G H^2\psi - \frac{1}{3}a^{-2}\nabla^2\Pi\right]$$

$$\delta q_{\mu\,G} = -2M_P^2\,\delta^i_\mu\left[\partial_i\left(\dot{\phi} + H\psi\right)\right]$$

$$\delta\pi_{\mu\nu\,G} = M_P^2\,\delta^i_\mu\delta^j_\nu\left[\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Pi\right]$$

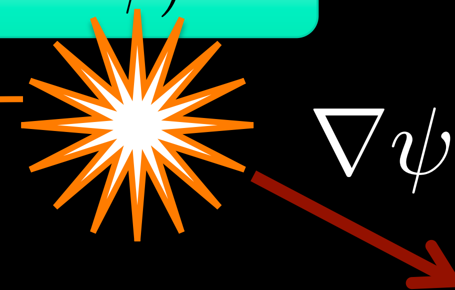
$$\Pi = \phi - \psi$$

We can always reconstruct an effective fluid EMT that gives the observed metric!

phenomenological parameters

$$ds^2 = -(1+2\psi)dt^2 + a(t)^2(1-2\phi)dx^2$$

a(t)



$\nabla\psi$

$\nabla_\perp(\phi+\psi)$

(lensing)

(velocity field)

observations probe space-time geometry
→ characterize geometry instead of fluid

deviations from "standard clustering":

$$-k^2\Psi \equiv 4\pi G a^2 \mu(a,\mathbf{k})\rho\Delta$$

$$\Phi \equiv \eta(a,\mathbf{k})\Psi$$

- extra clustering
- G_{eff}/G
- something else

We expect
μ = 1
η = 1
at low z

(many equivalent parametrisations cf e.g. MK 2012)

a hierarchy of DE modelling

more physical

fundamental action based models

effective field theories (action based)

equivalent fluid description

phenomenological metric parameters

cosmological observations

more general

model – EFT translation

Model Class		α_K	α_B	α_M	α_T
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_m)(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_m)(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$m^2(n_m+\kappa_\phi)/H^2M_{Pl}^2$	$m\kappa/HM_{Pl}^2$	0	0
galileon cosmology	[57]	$-3/2\alpha_M^3H^2r_c^2e^{2\phi/M}$	$\alpha_K/6-\alpha_M$	$-2\dot{\phi}/HM$	0
BDK	[26]	$\dot{\phi}^2K_{,\phi\phi}e^{-\kappa}/H^2M^2$	$-\alpha_M$	$\dot{\kappa}/H$	0
metric $f(R)$	[3, 72]	0	$-\alpha_M$	$B\dot{H}/H^2$	0
MSG/Palatini $f(R)$	[73, 74]	$-3/2\alpha_M^2$	$-\alpha_M$	$2\dot{\phi}/H$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot{\xi}}{M^2+H\dot{\xi}}$	$\frac{\dot{H}\dot{\xi}+H\ddot{\xi}}{H(M^2+H\dot{\xi})}$	$\frac{\ddot{\xi}-H\dot{\xi}}{M^2+H\dot{\xi}}$

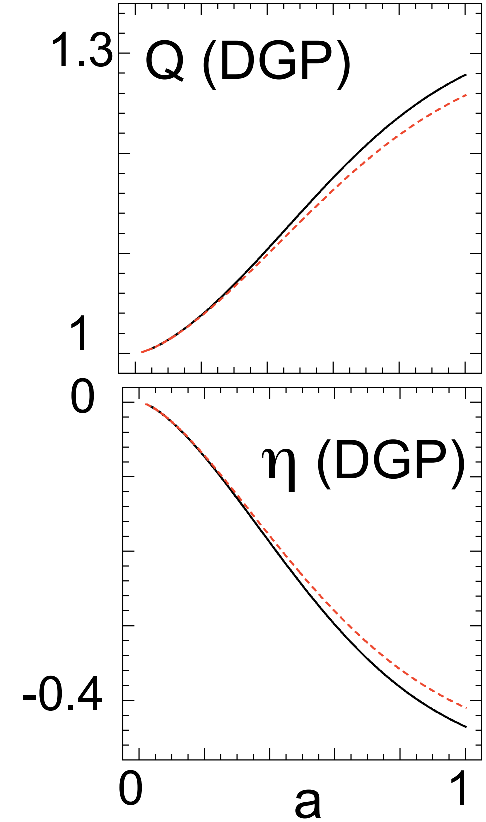
from Bellini & Sawicki, arXiv:1404.3713

model predictions for pheno

scalar field: $$S=\int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+V(\phi)\right)$$

One degree of freedom: $V(\phi)$ <-> $w(z)$ therefore other variables fixed: $c_s^2 = 1$, $\sigma = 0$
-> $\eta = 0$, $Q(k>>H_0) = 1$, $Q(k\sim H_0) \sim 1.1$

$$-k^2\Phi = 4\pi G a^2 Q\rho\Delta$$



(naïve) DGP: compute in 5D, project result to 4D

Lue, Starkmann 04
Koyama, Maartens 06

$$\eta=\frac{2}{3\beta-1}\qquad Q=1-\frac{1}{3\beta}$$

implies large 'DE' perturb.

Scalar-Tensor: Boisseau, Esposito-Farese, Polarski, Starobinski 2000, Acquaviva, Baccigalupi, Perrotta 04

$$\mathcal{L}=F(\varphi)R-\partial_\mu\varphi\partial^\mu\varphi-2V(\phi)+16\pi G^*\mathcal{L}_{\text{matter}}$$

$$\eta=\frac{F'^2}{F+F'^2}\qquad Q=\frac{G^*}{FG_0}\frac{2(F+F'^2)}{2F+3F'^2}$$

f(R): $S_g=\int d^4x\sqrt{-g}f(R)$ similar to scalar-tensor

???

Is it enough to say ‘my dark energy has $w(z) = \ldots$ ‘ when using the full Planck data?

A) I assume it’s a trick question, but why not?
B) No, I need to specify 1 extra quantity, namely ...
C) No, I need to specify 2 extra quantities, namely ...
D) It depends

only Λ has no perturbations

immediate consequences:

- dark energy is never completely smooth if $w \neq -1$ (and not even then if $\sigma \neq 0$!)
- for nearly all data sets we MUST give perturbation description, not just w
- sound horizons (and other things) lead to scale-dependent clustering

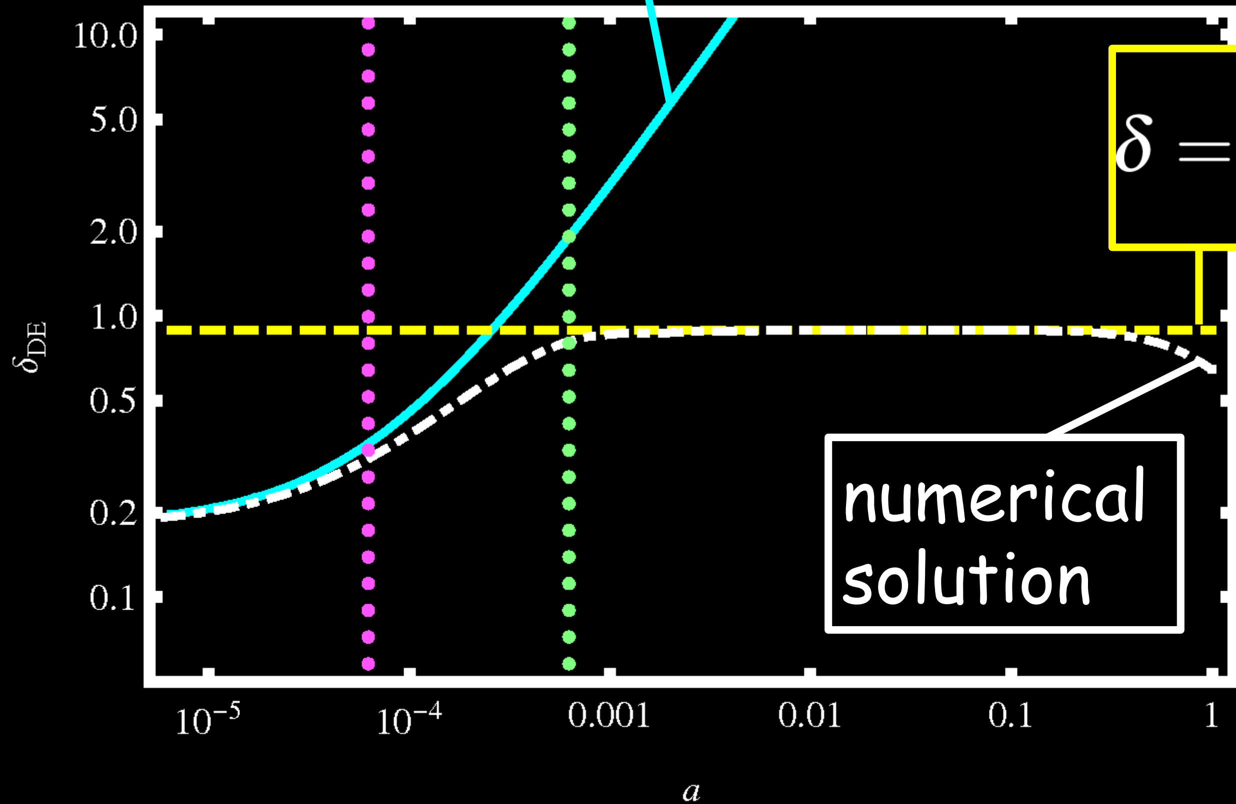
behaviour of scalar field δ

(e.g. Sapone & MK 09)

model $\{w, c_s, \sigma=0\}$; matter dom.: Φ = constant, $\delta_m \sim a$

$$\delta = \delta_0 (1+w) \left(\frac{a}{1-3w} + \frac{3H_0^2\Omega_m}{k^2} \right)$$

$\rightarrow$ $\delta(w=-0.8) \lesssim 1/20\ \delta(w=0)$ on subhorizon scales



$$\delta = \delta_0 \frac{3}{2} (1+w) \frac{H_0^2\Omega_m}{c_s^2 k^2}$$

- $w = -0.8$
- $c_s = 0.1$
- $k = 200\ H_0$

summary so far

- data requires some kind of dark energy
- cosmological constant fits, but is a bit unsatisfactory
- no other obvious natural fundamental theories
- so build effective theory that models d.o.f.
    - EFT – assumptions under control, but possibly limited
    - effective fluid – in the middle, can be linked to either
    - explicitly model geometry – fully general but may contain ‘impossible’ things
    - freedom in effective functions depends on physics that you want to model / test
- still need to find a fundamental theory
- non-perturbative / non-linear effects like screening
- and how about non-perturbative / non-linear aspects of GR?

LTB and Backreaction

Two large classes of models:

- Inhomogeneous cosmology: Copernican Principle is wrong, Universe is not homogeneous (and we live in a special place).

- Backreaction: GR is a nonlinear theory, so averaging is non-trivial. The evolution of the 'averaged' FLRW case may not be the same as the average of the true Universe.

testing the Copernican principle

1. Is it possible to test the geometry (Copernican principle) directly?
2. Yes! Clarkson et al, PRL (2008) -> in FLRW (integrate along ds=0):

$$H_0 D(z) = \frac{1}{\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k}\int_0^z \frac{H_0}{H(u)}du\right)$$

$$\Rightarrow H_0 D'(z) = \frac{H_0}{H(z)} \cos\left(\sqrt{-\Omega_k}\int_0^z \frac{H_0}{H(u)}du\right)$$

$$\rightarrow \left(HD'\right)^2 - 1 = \sin^2(\cdots) = -\Omega_k \left(H_0 D\right)^2$$

It is possible to reconstruct the curvature by comparing a distance measurement (which depends on the geometry) with a radial measurement of H(z) without dependence on the geometry.

Baryon Acoustic Oscillations may be able to do that
(or in future redshift drift or supernova dipole).

Lemaitre-Tolman-Bondi

do we live in the center of the world?

???

do we?

A) yes

B) no

Backreaction

normal approach: separation into “background” and “perturbations”

$$g_{\mu\nu}(t,x) = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(t,x)$$

$$\rho(t,x) = \bar{\rho}(t) + \delta\rho(t,x)$$

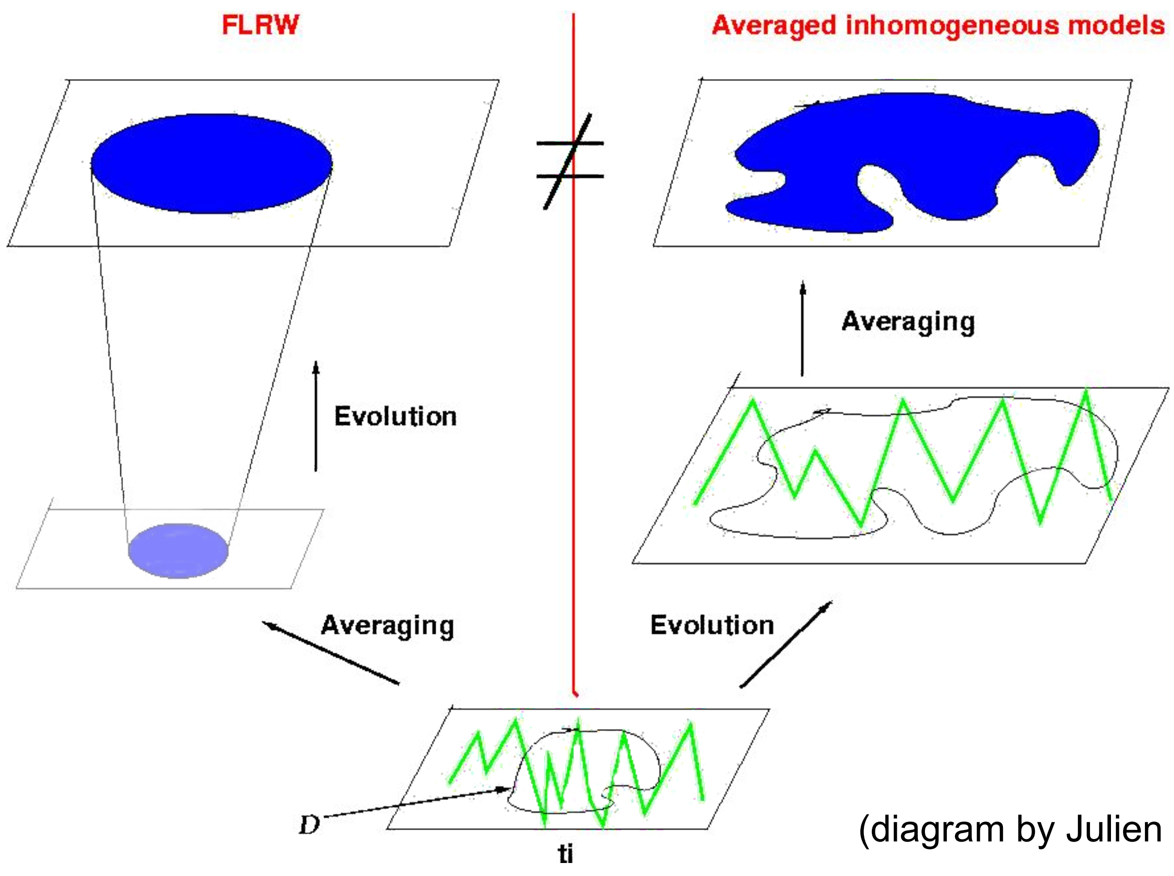
but which is the “correct” background, and why should it evolve as if it was a solution of Einsteins equations? The averaging required for the background does not commute with derivatives or quadratic expressions,

$$\partial_t \langle \phi \rangle \neq \langle \partial_t \phi \rangle \qquad \langle \theta^2 \rangle \neq \langle \theta \rangle^2$$

-> can derive set of averaged equations, taking into account that some operations not not commute: “Buchert equations”

average and evolution

the average of the evolved universe is in general not the evolution of the averaged universe!

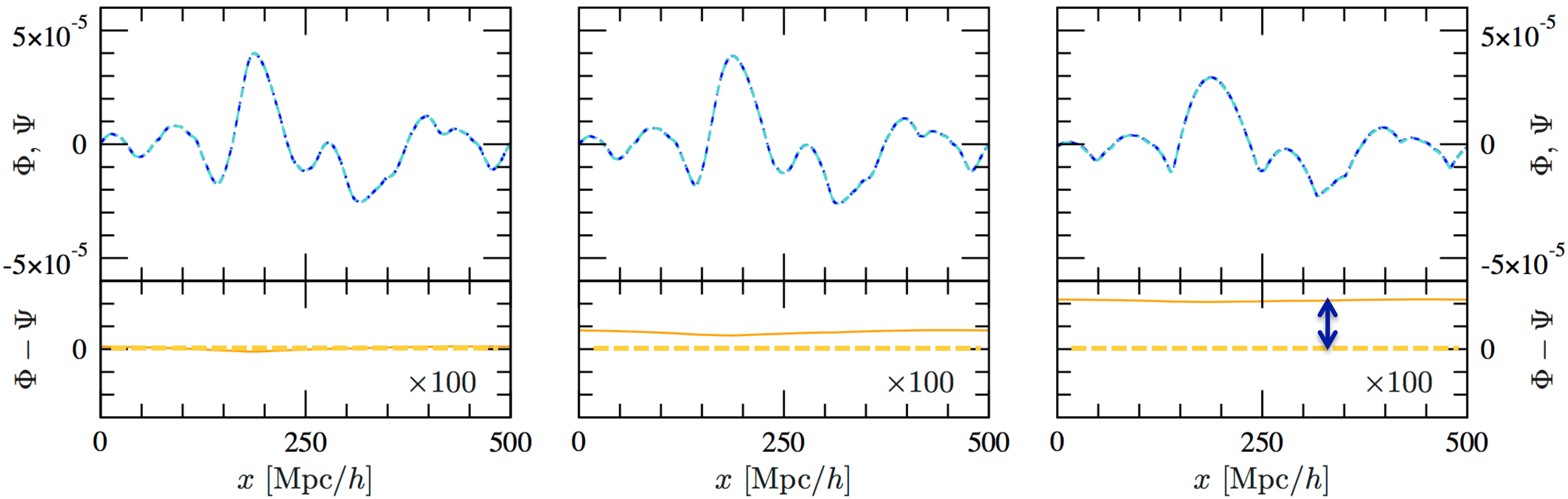


(diagram by Julien Larena)

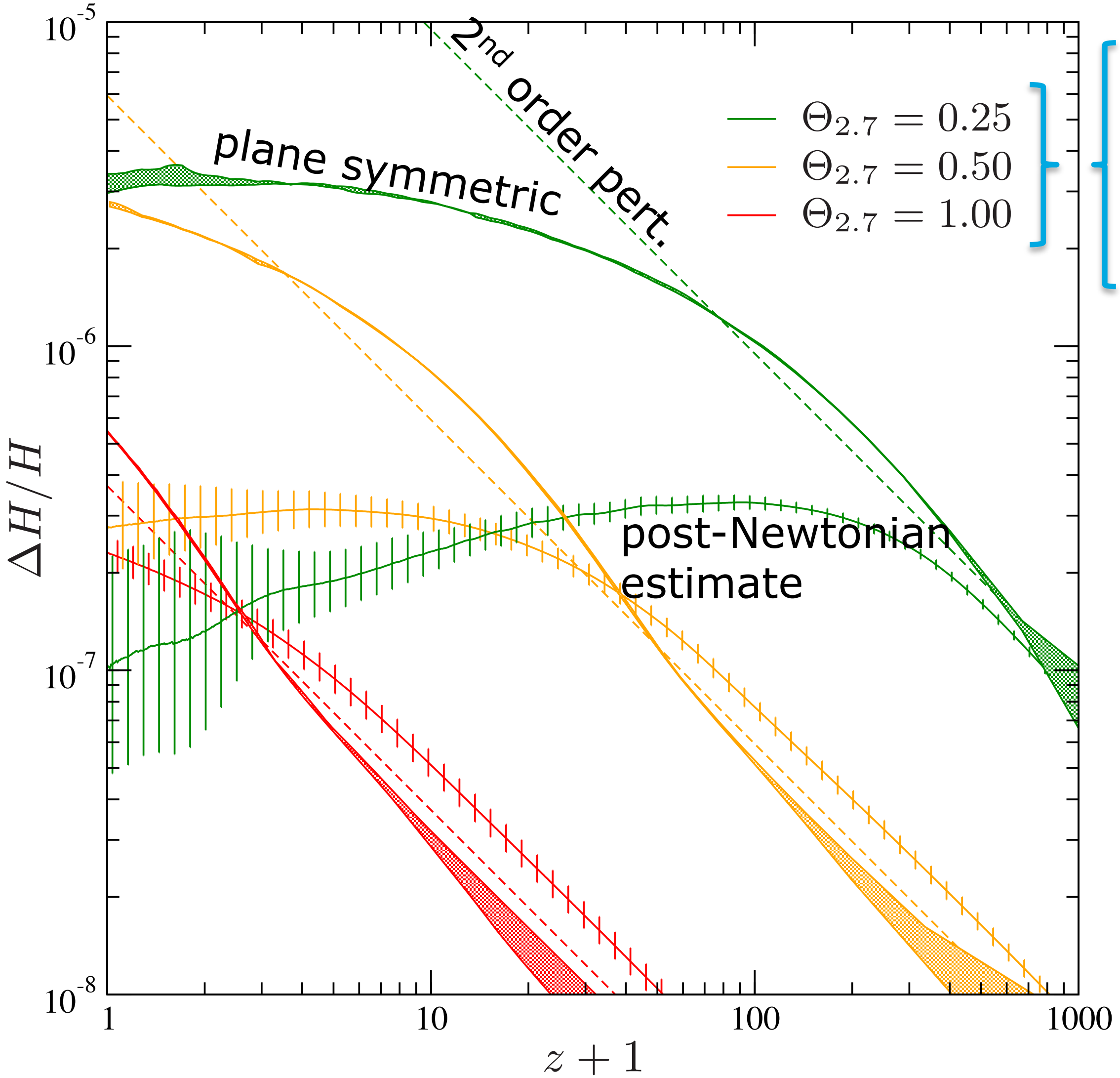
deviation from FLRW background in *gevolution*

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2$$

- absorb Ψ zero mode into time redefinition
- interpret Φ zero mode as correction to chosen background evolution a(t)
- can check if background evolves differently than in FLRW → not possible in Newtonian simulations!



backreaction seems to stop!



Earlier k_{eq} should increase effect (→ Clarkson & Umeh arXiv: 1105.1886)

True at early times, but correction stops increasing when density perturbations go non-linear!

(Perturbation theory diverges, doesn't help)

(new runs in progress)

Is backreaction self-limiting? Can we understand this?

Layzer-Irvine equation & virialization

correction to expansion rate from zero mode: $\mathcal{H} \to \mathcal{H} - \Phi_0' = n^\mu_{;\mu}/3$

equation for evolution of zero mode:

$$\boxed{2\Phi_0' + 3\mathcal{H}\Omega_m\Phi_0 = -\mathcal{H}\Omega_m \frac{T+U}{M}}$$

(In a 'Newtonian interpretation', using $2T = \Sigma m_i v_i^2$ and $2U = \Sigma m_i \psi(x_i)$)

Newtonian gravity:

Layzer-Irvine equation $T' + U' + \mathcal{H}\left(2T + U\right) = 0$

virialization: $2T = -U$

→ zero mode approaches a constant value $\boxed{\Phi_0 \to -(T+U)/(3M)}$

→ correction to expansion rate $\Delta\mathcal{H} = -\Phi_0'$
goes to zero in the virial limit!

(and relativistic corrections appear to be small)

brief survey

What do you think is the origin of ‘dark energy’?

A) cosmological constant
B) there is no dark energy, there is a problem with the data
C) there is no dark energy, there is a problem with our understanding of GR (eg backreaction)
D) a scalar-field like model (~ Horndeski/EFT)
E) something else
F) I don’t care, I want to go home!

DE theory summary

- the nature of dark energy is still unknown
- many models exist at level of action, including the cosmological constant
- also systematic and general frameworks exist
- key goal: test / exclude cosmological constant
- challenges especially in non-linear domain → advanced computational techniques & simulations
- important to keep an open mind for other possibilities (both DE/MG theories and especially wrong assumptions)